PHILOSOPHICAL TRANSACTIONS A

rsta.royalsocietypublishing.org

Research

Cite this article: Plotnitsky A. 2016 The future (and past) of quantum theory after the Higgs boson: a quantum-informational viewpoint. *Phil. Trans. R. Soc. A* **374**: 20150239. http://dx.doi.org/10.1098/rsta.2015.0239

Accepted: 22 February 2016

One contribution of 14 to a theme issue 'Quantum foundations: information approach'.

Subject Areas:
quantum physics

Keywords:
the Higgs boson, quantum mechanics, quantum field theory, quantum information theory, technology

Author for correspondence:
Arkady Plotnitsky
e-mail: plotnits@purdue.edu

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

The future (and past) of quantum theory after the Higgs boson: a quantum-informational viewpoint

Arkady Plotnitsky

Theory and Cultural Studies Program, Purdue University, West Lafayette, IN 47907, USA

Taking as its point of departure the discovery of the Higgs boson, this article considers quantum theory, including quantum field theory, which predicted the Higgs boson, through the combined perspective of quantum information theory and the idea of technology, while also adopting a *non-realist* interpretation, in 'the spirit of Copenhagen', of quantum theory and quantum phenomena themselves. The article argues that the 'events' in question in fundamental physics, such as the discovery of the Higgs boson (a particularly complex and dramatic, but not essentially different, case), are made possible by the joint workings of three technologies: experimental technology, mathematical technology and, more recently, digital computer technology. The article will consider the role of and the relationships among these technologies, focusing on experimental and mathematical technologies, in quantum mechanics (QM), quantum field theory (QFT) and finite-dimensional quantum theory, with which quantum information theory has been primarily concerned thus far. It will do so, in part, by reassessing the history of quantum theory, beginning with Heisenberg's discovery of QM, in quantum-informational and technological terms. This history, the article argues, is defined by the discoveries of increasingly complex configurations of observed phenomena and the emergence of the increasingly complex mathematical formalism accounting for these phenomena, culminating in the standard model of elementary-particle physics, defining the current state of QFT.

THE ROYAL SOCIETY PUBLISHING

© 2016 The Author(s) Published by the Royal Society. All rights reserved.

rsta.royalsocietypublishing.org Phil. Trans. R. Soc. A 374: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

1. Introduction

My starting point is the endpoint of the history to be traversed in this article—the discovery of the Higgs boson, arguably the greatest event of fundamental physics in the twenty-first century, and, thus far, a culminating event in the history of quantum physics. This event has been discussed at all levels and in all media, for example, quite adequately, in *Wikipedia*, with famous photographs of the events testifying to the discovery, nearly equally famous photographs of various components of the Large Hadron Collider (LHC) and the key relevant parts of the mathematical formalism of quantum field theory (QFT) [1,2]. These pictures are well known and easily located on the Web. I shall only cite the key relevant parts of the formalism, the epistemological nature of which will be discussed in §4:

> In the Standard Model, the Higgs field is a four-component scalar field that forms a complex doublet of the weak isospin SU(2) symmetry:
>
> $$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi^1 + i\phi^2 \\ \phi^0 + i\phi^3\end{pmatrix},$$
>
> while the field had charge $+1/2$ under the weak hypercharge U(1) symmetry (the convention where the electric charge, Q, the weak isopin, I_3, and the weak hypercharge, Y, are related by $Q = I_3 + Y$).
>
> The Higgs part of the Lagrangian is
>
> $$\mathcal{L}_H = \left|(\partial_\mu - igW_\mu^\alpha \tau^\alpha - i\frac{g'}{2}B_\mu)\right|^2 + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2,$$
>
> where W_μ^α and B_μ are the gauge bosons of the SU (2) and U (1) symmetries, and g and g' their respective coupling constant, $\tau^\alpha = \sigma^\alpha/2$ (where σ^α are Pauli matrices) a complete set of generators of the SU(2) symmetry, and $\lambda > 0$ and $\mu^2 > 0$, so that the ground state breaks the SU (2) symmetry. The ground state of the Higgs field (the bottom of the potential) is degenerate with different ground states related to each other by an SU(2) gauge transformation. It is always possible to pick up a gauge such that the ground state $\phi^1 = \phi^2 = \phi^3 = 0$. The expectation value of ϕ^0 in the ground state (the vacuum expectation value or vev) is then $\langle\phi^0\rangle = v/\sqrt{2}$, where $v = |\mu|/\sqrt{\lambda}$. The measured value of this parameter is $\sim$246 GeV/c^2. It has units of mass, and is the only free parameter of the Standard Model that is not a dimensionless number. Quadratic terms W_μ and B_μ arise, which give masses to the W and Z bosons:
>
> $$M_W = \frac{v\,|g|}{2},$$
> $$M_Z = \frac{\sqrt{g^2 + g'^2}}{2},$$
>
> with their ratio determining the Weinberg angle, $\cos\theta_W = M_W/M_Z = |g|/\sqrt{g^2 + g'^2}$, and leave a massless U(1) photon, γ. ([1], [3], pp. 690–700)

Now, what does all this (the photographs of the corresponding events, computer-generated images and data, staggering machinery of the LHC and the mathematics just described) mean and how is this possible? Without attempting to fully answer these questions, I consider a particular perspective on them and indicate some partial and possible answers that arise if one adopts this perspective. This perspective is defined by understanding quantum theory, from quantum mechanics (QM) to QFT to finite-dimensional quantum theory (QFDT), through the combined optics of quantum information theory and the concept of technology, in a particular, *non-realist*,

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

interpretation of these theories, in 'the spirit of Copenhagen', as W. Heisenberg aptly called it ([4], p. iv).[1] The discovery of the Higgs boson is the result of the joint workings of three technologies:

(1) the experimental technology of the LHC;
(2) the mathematical technology of QFT (sometimes coupled to the technology of philosophical thought); and
(3) digital computer technology.

I shall argue, however, that any quantum event is made possible through and is defined by the joint workings of the first two *types* of technology, with now the third increasingly involved at most levels, from the statistical analysis of the data considered to the digital modelling of the processes or theories involved (or even the digitalization of theories themselves), although digital technology will only be addressed in passing in this article. I am of course not suggesting (hence my emphasis on 'types') that all quantum events are reducible to those definable by any particular experimental technology or by QFT, but only that the first type of technology should be capable of detecting quantum events and the second of predicting them. Mathematical technology or the technology of philosophical thought are not conventional concepts, and they need to be explained, which I shall do later, for now using these terms in the general sense of technology as a tool or a set of tools that helps or enables us to do something—'to get from here to there'. Thus, the mathematics of QFT enables us to predict the Higgs boson and, with the help of other technologies just listed, to find it. There is yet another, more encompassing technology at work here: the overall technology of science as a cultural project or a set of such projects. This technology, however, will only be addressed in passing here, as my focus is on the relationships between mathematical and experimental technology in quantum theory, which radically reshaped these relationships as against classical physics or even relativity.

The first question I asked above, 'How is this possible?', brings one into the heart of the question of causality. Beginning with (and even before) Plato, our inquiry into Nature has always been a search for causes of things: 'Inquiry into nature is a search for the causes of each thing; why each thing comes into existence, why it goes out of existence, why it exists' ([5], §96 A6–10). This question could also be and has been asked in relation to the history of physics as a cultural technology. My aim here, however, is, again, to consider the possibility of such events in terms of quantum physics as such and the technologies forming it, which transformed our understanding of causality and, to begin with, reality, as against the classical view of both in philosophy and physics, from Plato or even the pre-Socratics on. In modern time, this view was codified by Kant. According to Kant: 'If, therefore, we experience that something happens, then we always presuppose that something else precedes it, which it *follows* in accordance with a rule' ([6], pp. 305, 308). This presupposition defines what may be called classical causality, and, in Kant and elsewhere, it is accompanied by and even arises from another presupposition, that of the possibility of forming a representation or at least a conception of the mechanism responsible for this rule, which presupposition defines realism. The question is, however, whether either presupposition is possible in quantum physics, given how its technologies—experimental, mathematical and philosophical (and by now even digital)—work there. To address this question, I shall consider the role of and the relationships among these technologies in QM, QFT and QFDT (the latter has thus far been the primary concern of quantum information theory). QFT splits into several theories: quantum electrodynamics (QED), now part of electroweak theory, and the theory of strong forces, quantum chromodynamics (QCD). These theories constitute the standard model of particle physics. The designation 'quantum theory' will refer to these theories.[2]

[1]This rubric is preferable to that of 'the Copenhagen interpretation', because there is no single such interpretation, even in the case of Bohr, who changed his view a few times.

[2]I shall only be concerned here with standard versions of these theories, beginning with standard QM (introduced by W. Heisenberg and E. Schrödinger in 1925–1926), as opposed to, for example, 'Bohmian mechanics', which is a mathematically different theory, rather than a different interpretation of QM. By 'quantum phenomena', I refer to those observable physical phenomena in considering which Planck's constant, h, must be taken into account, although, as will be seen below, these phenomena themselves may be described by means of classical physics. Quantum objects, which are responsible for the

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

The remainder of the article will proceed as follows. The next section, §2, outlines key concepts used in this article. Sections 3, 4 and 5 are devoted, respectively, to QM, QFT and QFDT, which have complex relations to each other and are marked as much by their differences as by their proximities. The final section, §6, offers a concluding reflection on, in M. Heidegger's phrase, 'the question concerning technology' in quantum physics [7].

2. An outline of concepts

I begin with the concepts of reality and realism, to which the concept of causality, mentioned above and discussed below, is fundamentally related. These concepts could be defined in different ways, and they have been debated from the pre-Socratics on, in modern time especially following D. Hume (who offered the first modern critique of causality) and Kant, and with a new vigour in recent decades.[3] The definitions given here cannot capture all concepts thus designated, but they are sufficient for my purposes.

By 'reality' I refer, very generally, to that which exists or is assumed to exist, without making any claims concerning the character of this existence. I understand existence as the capacity to have effects upon the world with which we interact, the world that has such effects upon itself. In the case of physics, it is Nature or matter, which is usually assumed to exist independently of our interactions with it, and to have existed when we did not exist and to continue to exist when we will no longer exist. This assumption also holds in non-realist interpretations of quantum theory, in the absence of any representation of the character of this existence. The existence and hence *reality* of quantum objects, thus placed beyond representation or even conception, is inferred from effects they have on our world, specifically experimental technology.

I define realism as, in each given realist theory, a specific set of claims concerning what exists and, especially, *how* it exists, provided by the corresponding theory. In this definition, any form of realism is more than only a claim concerning the existence, *reality*, of something, such as physical objects. Realism is defined primarily by claims concerning the *character* of this existence. Realist theories are sometimes also called ontological theories. The term 'ontological' may carry additional philosophical connotations, with which I shall not be concerned here. Accordingly, the terms 'realism' and 'ontology' will, unless qualified otherwise, be used interchangeably. Realist theories may be divided into the following two types.

According to *the first type of realism*, a realist theory would offer a representation, typically a mathematically idealized representation by means of the corresponding model, of the objects or systems considered and their behaviour, or sometimes, as in the so-called structural realism, of the structures defining such systems and their behaviour. The properties and the relationships among them thus represented are usually assumed to exist independently of our interaction with the objects in question. The mathematical formalism of a given realist theory of this first type comprises a *representational* mathematical 'model' of reality. Realist theories of the second type do not contain such models.

The second type of realism would presuppose an independent architecture (which may be temporal) of reality governing the behaviour of the ultimate objects considered, even if this architecture cannot be represented, even ideally, either at a given moment in history or perhaps ever, but, if so, only due to practical limitations. In the first of these two eventualities, a theory that is merely predictive may be accepted for the lack of a realist alternative, but under the assumption

appearance of quantum phenomena, could be macroscopic, although their quantum nature would be defined by their ultimate microscopic constitution and hence by the role of h in the corresponding phenomena. By 'quantum physics', I refer to the totality of quantum phenomena and quantum theories. The terms 'classical phenomena', 'classical mechanics', 'classical theory' and 'classical physics' will be used in parallel.

[3] I am referring to such authors as T. Kuhn, I. Lakatos and their followers in the so-called constructivist studies of science. The literature on these subjects, both more traditional and more revisionist, is extensive. To give a few representative references, for analytic–philosophical approaches, see [8,9]; for more restrained post-Kuhnian approaches, see [10,11]; and in the context of the relationships between classical and quantum physics ([12], pp. 177–233] and for more expressly constructivist treatments, see [13,14]. Some of these works explore what I refer to above as the technology of science as a cultural project or a set of such projects, and the last two expressly deal with technology, including the experimental technology of particle physics [13].

rsta.royalsocietypublishing.org Phil. Trans. R. Soc. A 374: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

or with a hope that a future theory will do better, in particular by being a realist theory of the first, representational, type. Einstein adopted this attitude toward QM, which he expected to be eventually replaced by such a theory. In general, even in the second eventuality, the ultimate constitution of Nature is customarily deemed to be conceivable following the realist models of classical physics.

What thus unites both conceptions of realism and defines realism most generally is the assumption that an *architecture of reality*, rather than only reality itself, exists independently of our interactions with it, or at least that the concept of architecture or structure would apply to reality. In other words, realism is defined by the assumption that the ultimate constitution of nature possesses attributes and relationships among them that may be either (i) known in one degree or another and, hence, represented, at least ideally, by a theory or model or (ii) unknown or even unknowable, corresponding to the two types of realism just defined.

Non-realist interpretations of quantum phenomena and QM, at least those in the spirit of Copenhagen (the only non-realist interpretations considered here), not only do not make any of these realist assumptions, but also, in Bohr's language, '*in principle* [exclude]' them ([15], v. 2, p. 62). In this view, quantum objects exist, are *real*, but the character of their existence is such that it '*in principle* exclude[s]' a realist mathematical model representing, even ideally, their behaviour. There is no story to be told and no conception to be formed about how quantum events come about. Non-realist interpretations of QM imply that only predictions, of a probabilistic or statistical nature, concerning quantum phenomena are possible, even as concerns elemental individual quantum processes, such as those associated with elementary particles (hereafter referred to as 'elementary quantum processes'). This is one of the reasons for such interpretations, because this character of quantum predictions is an experimental fact, given that the repetition of identically prepared experiments, in general, leads to different outcomes. Unlike in classical physics, this difference cannot be diminished beyond a certain limit (defined by Planck's constant, h) by improving the conditions of measurement, a fact also reflected in the uncertainty relations, which are correlative to the statistical nature of quantum predictions.

There is still a form of realism associated with non-realist interpretations of quantum theory, at least those considered here, beginning with that of Bohr. It is defined by the interpretation of the physics of measuring instruments in which the outcomes of quantum experiments are registered as the effects of the interaction between quantum objects and these instruments. These instruments, or rather *their observable parts*, are assumed to be described by classical physics, which, however, cannot predict these effects. On the other hand, the interaction between quantum objects and measuring instruments is quantum, and, hence, it is not amenable to a realist treatment. In each single experiment, however, this interaction leaves, as its effect, a trace, a mark or set of marks in a measuring instrument, both of which could be very complex, as they are in the case of the photographic traces and experimental technology of the Higgs boson. The numerical data associated with such marks can be predicted in the probabilistic or statistical terms by QM or, in high-energy regimes, by QED. The quantum *origins* of this 'trace' are beyond the reach of experiment or (in terms of representation) theory and possibly thought in general. Such marks, however, or the data associated with them can be treated by means of a representational account and made part of a permanent record, which can be unambiguously defined, communicated, and so forth, and in this sense is objective. The available configurations of these marks may compel one to introduce non-realist interpretations of quantum objects and their behaviour, as responsible for the appearance of these marks as effects of these processes, as unavailable to representation by means of QM or otherwise, or even conception.

The lack of causality, *as it is classically understood*, say, again following Kant, is an automatic consequence of such interpretations, especially if one places the reality of quantum objects and processes beyond conception, because causality would imply at least a partial conception of this reality. However, even if one adopts a weaker assumption, which only precludes a representation of this reality, causality is difficult to maintain because to do so one requires a sufficient representation, analogous to that found in classical physics. Schrödinger expressed this

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

difficulty in his cat-paradox paper: 'if a classical state does not exist at any moment, it can hardly change causally' ([16], p. 154).

I shall now define classical causality. Given, however, that most of my uses of the term refer to classical causality, by 'causality' I shall mean primarily classical causality, unless qualified otherwise. I shall indicate a few alternative conceptions of causality, which are applicable in quantum theory in the absence of classical causality. If understood classically, causality is an ontological category, part of reality. It relates to the behaviour of physical systems whose evolution is defined by the fact that the state of a given system (as idealized by a given theory or model) is determined at all moments of time by its state at a particular moment of time, indeed at any given moment of time. This concept is in accord with Kant's principle of causality cited above and has been commonly used since, which states that, if an event takes place, it has a determinable cause or set of causes of which this event is an effect. It is also commonly (although not universally) assumed that the cause must be antecedent to, or at least simultaneous with, the effect. Quantum phenomena, at least in non-realist interpretations, violate the principle of (classical) causality, because no determinable event could be established as the cause of a given event, and only statistical correlations between certain events could be ascertained.

I define 'determinism' as an epistemological category, part of our knowledge of reality. Determinism denotes our ability to predict the state of a system, at least, again, as defined by an idealized model, exactly, rather than probabilistically, at any moment of time once we know its state at a given moment of time. Just as in the case of causality, this category, too, may be designated as classical determinism, because one could in principle define determinism differently. Once again, however, unless qualified otherwise, I shall by determinism refer to classical determinism. Determinism is sometimes also used in the same sense as causality, and in the case of classical mechanics, which deals with single objects or a small number of objects, causality and determinism coincide. Once a system is sufficiently large, one needs a superhuman power to predict its behaviour exactly, as was famously explained by P. S. Laplace, who invented the figure of Laplace's demon as an image of this power.

While, however, it follows automatically that non-causal behaviour, *considered at the level of a given model*, cannot be handled deterministically, the reverse is not true. The qualification is necessary because we can have causal models of processes in Nature that may not ultimately be causal. The fact that the causal models of classical physics apply and are effective within the proper limits of classical physics does not mean that the ultimate character of the actual processes that are responsible for classical phenomena are causal. They may not be, for example, by virtue of their ultimately quantum nature. Nor, conversely, does the non-causal character of a model, for example that of QM, guarantee that quantum behaviour is non-causal. It may ultimately prove to be causal and, to begin with, amenable to a realist treatment. Rigorously, quantum phenomena only preclude determinism, because, as noted earlier, identically prepared quantum experiments in general lead to different outcomes. While individual quantum experiments are repeatable in terms of the state of measuring instruments before they are performed, they are not repeatable as concerns their outcomes. Only the statistics of repeated experiments are repeatable. It would be difficult to do science without being able to repeat at least the statistical data our experiments provide. However, it is quite possible to do science if one can only repeat the statistical data thus obtained. The lack of causality or, to begin with, of realism in the corresponding interpretations of quantum phenomena and QM are, as I said, *interpretive inferences* from this situation.[4]

I shall now define some alternative conceptions of causality. In particular, the term 'causality' is often used in accordance with the requirements of (special) relativity, which restricts causes to those occurring in the backward (past) light cone of the event that is seen as an effect of this cause, while no event can be a cause of any event outside the forward (future) light cone of that event. In other words, no physical causes can propagate faster than the speed of light in

[4]Accordingly, such interpretations do not exclude the possibility of causal or realist interpretations of QM, or alternative causal or realist quantum theories, such as Bohmian mechanics (which is non-local), or theories defined by an assumption of a deeper underlying causal dynamics, which makes QM an 'emergent' theory. Among recent proposals along these lines is A. Khrennikov's 'pre-quantum classical statistical field theory' [17,18].

rsta.royalsocietypublishing.org Phil. Trans. R. Soc. A 374: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

a vacuum, *c*, which requirement also implies temporal locality. Technically, this requirement as such only *restricts* classical causality, by a relativistic antecedence postulate, rather than precludes it, and relativity theory itself, special or general, is (locally) a classically causal and even, ideally, deterministic theory. QM, as a probabilistic or statistical theory of quantum phenomena, lacks classical causality, at least in non-realist interpretations. However, QM, QFT or QFDT could be said to respect a form of local 'causality' as concerns its probabilistic or statistical predictions, which are assumed to conform to both temporal and spatial locality, and hence the relativistic antecedence, just described. (I shall discuss locality as such below.) Thus, compatibility with relativistic requirements would be maintained insofar as an already performed experiment determines, probabilistically or statistically, a possible outcome of a future experiment, without assuming classical causality. More generally, whatever *actually happens* is only defined by spatially and temporally local factors, although the probabilistic or statistical *predictions*, while always made locally in physical terms, need not be local in quantum theory and could concern distant events, as in situations of the EPR (Einstein–Podolsky–Rosen) type.[5]

By 'randomness' or 'chance', I refer to a manifestation of the unpredictable. Randomness and chance are not the same, but the difference between them is not germane to my argument. It may or may not be possible to estimate whether a random event would occur, or even to anticipate it as an event. A random event may or may not result from some underlying causal dynamics unavailable to, or assumable by, us. The first eventuality defines what may be called classical randomness, an appearance of randomness underlain by a hidden causal process. This view has been a dominant form of realist thinking throughout the history of Western thought, from the pre-Socratics on. Thus, in classical statistical physics, randomness and the resulting recourse to probability are due to insufficient information concerning systems that are at bottom causal but whose mechanical complexity prevents us from accessing their causal behaviour and making deterministic predictions concerning this behaviour. The situation in quantum physics is different, given the difficulties of sustaining arguments for the causality of the independent behaviour of quantum objects, even elemental individual quantum objects, such as elementary particles. If an interpretation is non-realist, the absence of causality is, again, automatic, and the recourse to probability or statistics is unavoidable in principle, even in the case of elemental quantum processes and events. According to Bohr:

> [I]t is most important to realize that the recourse to probability laws under [quantum] circumstances is essentially different in aim from the familiar application of statistical considerations as practical means of accounting for the properties of mechanical systems of great structural complexity. In fact, in quantum physics we are presented not with intricacies of this kind, but with the inability of the classical frame of concepts to comprise the peculiar feature of indivisibility, or 'individuality,' characterizing the elementary processes. ([15], v. 2, p. 34)

Probability and statistics deal with estimates of the occurrences of certain individual or collective events, which defy deterministic handling (whether there is or not a hidden underlying causality determining these events), in physics in accordance with mathematical probability theories. The terms 'probabilistic' and 'statistical' are used differently. 'Probabilistic' refers to our estimates of the probabilities of either individual or collective events, such as that of a coin toss or of finding a quantum object in a given region of space. 'Statistical' refers to our estimates concerning the outcomes of identical or similar experiments, such as that of multiple coin-tosses or repeated identically prepared experiments with quantum objects, or to the average behaviour of certain objects or systems.[6] A given definition of probability may already reflect this difference, as in the case of the Bayesian versus the frequentist understanding of probability. The Bayesian

[5] For discussions of quantum forms of causality, see [19,20]. See [21] for a concept of quantum information causality (cf. also [22], p. 421]).

[6] The standard use of the term 'quantum statistics' refers to the behaviour of large multiplicities of identical quantum objects, such as electrons and photons, which behave differently, in accordance with, respectively, the Fermi–Dirac and the Bose–Einstein statistics.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

understanding defines probability as a degree of belief concerning a possible occurrence of an individual event on the basis of the relevant information we possess. This makes the probabilistic estimates involved, generally, subjective, although there may be an agreement (possibly among a large number of individuals) concerning such estimates. The frequentist understanding, also referred to as 'frequentist *statistics*', defines probability in terms of sample data by an emphasis on the frequency or proportion of these data, which is more objective. In quantum physics, where, as explained above, exact predictions appear to be, in general, impossible even in dealing with primitive individual processes and events, one considers identical quantum objects such as electrons or photons (not the identical preparation of each, which cannot be ensured), and the identically prepared measuring instruments as the initial condition of repeated experiments. The identical preparation of the instruments can be controlled because their observable parts can be described classically, while that of quantum objects themselves, again, cannot be. This is why the outcomes of quantum experiments will in general be different. This situation could, however, be interpreted on either frequentist or Bayesian lines, even if one adopts a non-realist view [18].

This summary sidesteps some of the deeper aspects of probability, but it suffices for my purposes.[7] I would like to add that probability introduces an element of order into situations defined by the role of randomness in them, and enables us to handle such situations better. Probability or statistics is, thus, about the interplay of randomness and order. This fact takes on a unique significance in quantum physics because of the presence of statistically ordered correlations (not found in classical physics) between certain data, such as those of the experiments of the EPR type. One of the greatest mysteries, if not the greatest mystery, of quantum physics is that, under certain, but not all, circumstances, random individual events conspire to form statistically correlated and thus statistically ordered multiplicities. These correlations are correctly predicted by QM. This, again, does not mean that it is the only formalism that can predict these correlations. Bohmian mechanics, the predictions of which coincide with those of standard QM, predicts them, but at the cost of non-locality.

The principle of locality states that no instantaneous transmission of physical influences between spatially separated physical systems ('action at a distance') is allowed or, which is a more current formulation, that physical systems can only be physically influenced by their immediate environment. Although not strictly equivalent, these two formulations are equivalent in most contexts (including those considered in this article). Non-locality in this sense is usually (although not always) seen as undesirable. Standard QM appears to avoid it. As noted above, under certain circumstances, such as those of the EPR-type experiments, QM can make *predictions* concerning the state of spatially separated systems, while allowing one to maintain that the physical circumstances of making these predictions and verifying them are *local*. However, the question of the locality of QM or quantum phenomena is a matter of great subtlety and much controversy, especially in the wake of the Bell and Kochen–Specker theorems and related findings. These developments even led to the dominance of the question of locality in recent debates concerning quantum theory, although the question of realism has continued to remain germane, in part given the lack of realism as a possible alternative to non-locality.[8]

The concept or principle of locality is commonly associated with relativity. However, the concept of locality, including as implied by Einstein's arguments (based on prohibiting an instantaneous, 'spooky', action at a distance) concerning the EPR-type experiments, is independent of other keys concepts with which it is linked in relativity, in particular the Lorentz invariance of special relativity. Technically, relativity prohibits an instantaneous propagation or transmission of physical influences not instantaneously but faster than the (finite) speed of light in a vacuum, a requirement that could, in principle, be violated, while still allowing for locality, as a different speed limit on physical action may emerge. The standard QM is not relativistic, but it is, or may be interpreted as, local, or in any event may be required to be local. This need not mean

[7]See [23–25] and references therein. On the Bayesian philosophy of probability, in two different versions of it, see [26,27].

[8]The literature dealing with these subjects is nearly as immense as that on interpretations of QM. Among the standard treatments are [28–30]. See also [31], for a current assessment of Bell's theorem. These theorems and most of these findings pertain to quantum data and, thus, do not depend on QM.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

that the locality principle is necessarily quantum in origin. It may, however, reflect deeper aspects of Nature than those captured by relativity theory.

3. Heisenberg's quantum mechanics as, and not as, quantum information theory

Heisenberg's initial approach to QM and Bohr's interpretation of the theory, first offered in 1927, was defined by the following main principles:

(1) the principle of quantum discreteness or the QD principle, according to which all quantum phenomena, defined as what is observed in measuring instruments, are individual and discrete, which is not the same as the (Democritean) atomic discreteness of quantum objects themselves;[9]
(2) the principle of the probabilistic or statistical nature of quantum predictions, even in the case of quantum processes and events associated with elemental individual constituents of nature, the QP/QS principle; and
(3) the correspondence principle, which, as initially used by Bohr and others, required that the predictions of quantum theory must coincide with those of classical mechanics at the classical limit, but was given a mathematical form by Heisenberg, which required that both the equations and variables used convert into those of classical mechanics in the classical limit.

In Heisenberg, each of these principles also gives rise to a mathematically expressed postulate. This is crucial if one wants, on the basis of a given set of principles, to establish quantum theory (QM, QFT or QFDT) as a mathematical model predicting the outcome of quantum experiments, which is all one needs by the QP/QS principle. The QD principle originated in Bohr's 1913 theory of the hydrogen atom, as based on 'quantum postulates', pertaining to the discrete behaviour ('quantum jumps') of electrons in atoms. According to Heisenberg:

> In order to characterize this radiation we first need the frequencies which appear as functions of two variables. In quantum theory these functions are in the form:
>
> $$v(n, n-\alpha) = 1/h\{W(n) - W(n-\alpha)\}. \quad \text{([32], p. 263)}$$

The postulate that mathematically expressed the QP/QS principle in Heisenberg was in effect the formula for the probability amplitudes-cum-the equivalent of Born's rule for quantum probability. Born's rule is also a postulate, as reflected in related uses of the term in Neumann's projection postulate or Lüder's postulate. Heisenberg only formulated this postulate in the particular case of quantum jumps and the hydrogen spectra, rather than, as Born did, as universally applicable in QM. Born's rule is not inherent in the formalism but is added to it, as postulated independently.

The correspondence principle played an essential role in the development of matrix mechanics by Heisenberg, who, however, gave it a new, mathematical form, made it 'the mathematical correspondence principle'. In this form, the principle required that both the equations of QM (which were formally those of classical mechanics) and the variables used (which were different) convert into those of classical mechanics at the classical limit. (The processes themselves, however, are still assumed to be quantum.) This definition converts the correspondence principle into a mathematically expressed postulate. The principle acquires its own form in QED and QFT, insofar as the equations and variables used by the theory must convert into those of QM in the

[9]In the 1930s, Bohr, building on the understanding that quantum objects and their behaviour cannot be observed independently of their interactions with measuring instruments, rethought quantum discreteness in terms of his concept of phenomenon, 'the observations obtained under specified circumstances, including an account of the whole experimental arrangement' ([15], v. 2, p. 64).

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

corresponding limits. In Heisenberg's case, the equations used were formally the same as those of classical mechanics, which made this part of the conversion (but of course not the conversion of variables) automatic.

Bohr's interpretation, first proposed in 1927, added the complementarity principle:

(4) the principle, which stems from the concept of complementarity, requires: (i) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet (ii) the possibility of applying each one of them separately at any given moment of time; and (iii) the necessity of using all of them at different moments for a comprehensive account of the totality of phenomena that one must consider in quantum physics.[10] It is difficult to express complementarity in a mathematical postulate. However, the non-commutativity of the quantum-mechanical formalism may be seen as the mathematical expression of complementarity.

I shall now consider Heisenberg's discovery of matrix mechanics, as it appears in his original paper. Bohr's initial comments on matrix mechanics in 1925 (before Schrödinger's version was introduced) shows a clear grasp of Heisenberg's approach:

> In contrast to ordinary mechanics, the new quantum mechanics does not deal with a space–time description of the motion of atomic particles. It operates with manifolds of quantities which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quantum theory]. ([15], v. 1, p. 48)

This approach may be considered in quantum-informational terms. The experimental situation is defined by (i) certain *already obtained* information, concerning the energy of an electron, derived from spectral lines (associated with a hydrogen atom) observed in measuring instruments; and (ii) certain possible future information, concerning the energy of this electron, *to be obtainable* from spectral lines, predictable in probabilistic or statistical terms, again associated with events to be observed in measuring instruments. Heisenberg's strategy was to abandon the task of developing a mathematical scheme representing how these data or information are connected by a spatio-temporal process, and derive his predictions from this scheme. He knew from the old quantum theory that this would be difficult and perhaps impossible to do, especially given that even an elementary individual event, such as the emission of a single photon, could not be predicted exactly, even ideally. Instead, he decided to try to find a mathematical scheme that would just enable these predictions, using the principles stated above (except for complementarity, which came later). Note also that unlike in Bohr's atomic theory, in Heisenberg's scheme, the stationary states were no longer represented in terms of orbital motion, but only in terms of energy values, which would change discontinuously and acausally, changes statistically predicted by Heisenberg's scheme. Part of the formal mathematical architecture of the scheme was provided by the equations of classical mechanics, formally adopted by Heisenberg, by the mathematical correspondence principle. Classical variables, however, would not give correct predictions, and had to be replaced by new variables, which were complex-valued matrix variables (essentially operators in Hilbert spaces over complex numbers) of a type never used in physics previously, but which would convert into classical variables (functions of real variables) in the classical limit.

Two qualifications are in order. First, I am not saying that Heisenberg's matrix mechanics was, or that QM or QFT *is*, (only) quantum information theory. Heisenberg was concerned with how fundamental quantum objects and processes work, even though these workings defy being represented, which fact is an effect of these workings. The information at stake was still about them, rather than was part of information processing or communication by using quantum

[10] The concept of complementarity, thus formulated, reflects Bohr's later works, from 1929 on, where the concept is exemplified, in particular, by the position and the momentum measurements, thus correlatively to the uncertainty relations. See [33] for the development of Bohr's views and changes in his interpretation.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

technology, experimental and mathematical. But then, as will be seen in §5, quantum information theory, too, may serve the purposes of fundamental physics, rather than only aim at theorizing quantum information processing between devices. My point is that, in Heisenberg's approach, QM contains a constitutive quantum-informational structure within it.

My second qualification follows Heisenberg's own:

> It should be distinctly understood, however, this [the deduction of the fundamental equation of quantum mechanics] cannot be a deduction in the mathematical sense of the word, since the equations to be obtained form themselves the *postulates* of the theory. Although made highly plausible by the following considerations [given by Heisenberg], their ultimate justification lies in the agreement of their predictions with the experiment. ([4], p. 108)

This is an important point in the context of projects aimed at deriving quantum theory from fundamental principles. It opens the question concerning which postulates are considered 'natural' or 'reasonable', or what constitutes a proper derivation. One might contend that it is not sufficiently first-principle-like to see the equation of QM as a postulate, and one might also prefer a less mixed derivation of it than that of QM by Heisenberg or Schrödinger and throughout its history. So far, these attempts, such as those discussed in §5, have concerned QFDT. This might, however, be changing [34].

In reflecting on Heisenberg's conception of these quantities, one might observe first that, in order to invent a new concept of any kind, one has to construct a phenomenological entity or a set of entities and relations among them. In physics, one must give this construction a mathematical architecture, defining the corresponding mathematical model, with which one might start, as Heisenberg in fact did, and which enables the theory to *relate* to observable phenomena and measurable quantities associated with these phenomena. Heisenberg's invention of his matrices was made possible by his idea of arranging algebraic elements corresponding to numerical quantities (transition probabilities) into infinite square tables. It is true that, once one deals with the transitions between two stationary states, rather than with a description of such states, matrices appear naturally, with rows and columns linked to each possible state, respectively. This naturalness, however, appeared or *became natural* only in retrospect. This arrangement was a phenomenological construction, which amounted to that of a mathematical object, a matrix, an element of a general non-commutative mathematical structure, part of (infinite-dimensional) linear algebra, in effect in a Hilbert space over complex numbers, in which Heisenberg's matrices ('observables') form an operator algebra. One can also see it as a representation of an abstract algebra, keeping in mind that Heisenberg's infinite matrices were unbounded, which fact is necessary to have the uncertainty relations for the corresponding continuous variables. As unbounded self-adjoint matrices, these matrices do not form an algebra with respect to the composition as a non-commutative product, although some of them satisfy the canonical commutation relation.

Heisenberg begins his derivation by observing that 'in quantum theory it has not been possible to associate the electron with a point in space, *considered as a function of time*, by means of observable quantities. However, even in quantum theory it is possible to ascribe to an electron the emission of radiation' ([32], p. 263; emphasis added). My emphasis reflects the fact that in principle a measurement could associate an electron with a point in space, but not as a function of time, as in classical mechanics. Heisenberg then says:

> In order to characterize this radiation we first need the frequencies which appear as functions of two variables. In quantum theory these functions are in the form:
>
> $$v(n, n-\alpha) = 1/h\{W(n) - W(n-\alpha)\} \quad (1)$$
>
> and in classical theory in the form
>
> $$v(n, \alpha) = \alpha v(n) = \alpha/h(dW/dn). \quad ([32],\ \text{p. } 263)$$

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

This difference, which reflects the QD principle, leads to a difference between classical and quantum theories as concerns the combination relations for frequencies, which, in quantum theory, correspond to the Rydberg–Ritz rules. However, 'in order to complete the description of radiation [in accordance, by the mathematical correspondence principle, with the classical Fourier representation] it is necessary to have not only frequencies but also the amplitudes' ([32], p. 263). On the one hand, then, by the correspondence principle, the new, quantum-mechanical equations must contain amplitudes. On the other hand, they could no longer serve their classical physical function (as part of a continuous representation of motion) and are instead related to the discrete transitions between stationary states. In Heisenberg's theory and in QM since then, these 'amplitudes' become no longer amplitudes of physical motions, which makes the name 'amplitude' itself an artificial, *symbolic* term. They are instead linked to the probabilities of transitions between stationary states: they are what we now call probability amplitudes.[11] The corresponding probabilities are derived by a form of Born's rule for this case. The standard rule for adding the probabilities of alternative outcomes is changed to adding the corresponding amplitudes and deriving the final probability by squaring the modulus of the sum. The mathematical structure thus emerging is in effect that of vectors and (in general, non-commuting) Hermitian operators in complex Hilbert spaces, which are infinite-dimensional, given that we deal with continuous variables. Heisenberg explains the situation in these terms in his 1930 book, with a fully fledged QM in hand ([4], pp. 111–122). In his original paper, he argues as follows:

> The amplitudes may be treated as complex vectors, each determined by six independent components, and they determine both the polarization and the phase. As the amplitudes are also functions of the two variables n and α, the corresponding part of the radiation is given by the following expressions:
>
> Quantum-theoretical:
>
> $$\mathrm{Re}\{A(n, n-\alpha)\mathrm{e}^{i\omega(n,n-\alpha)t}\}$$
>
> Classical:
>
> $$\mathrm{Re}\{A_{\alpha}(n)\mathrm{e}^{i\omega(n)\alpha t}\}. \quad ([32],\ \text{p. } 263)$$

The problem—a difficult and, 'at first sight', even insurmountable problem—is now apparent: '[T]he phase contained in A would seem to be devoid of physical significance in quantum theory, since in this theory frequencies are in general not commensurable with their harmonics' ([32], pp. 263–264). This incommensurability, which is in an irreconcilable conflict with classical electrodynamics, was one of the main consequences of Bohr's 1913 atomic theory [35], on which Heisenberg builds. Heisenberg now proceeds to inventing a new theory around this problem, in effect, by making it into a solution, saying: 'This is not a problem, the classical way of thinking is'.

[11] It is true that quantum data may present themselves in terms of interferometry, which is seen in the graphical representation of counting rates (proportional to the probabilities in question) that are typically oscillatory. In referring to these data, one could speak more intuitively, albeit still metaphorically, of 'amplitudes' of these oscillations, just as one speaks of 'interference' in referring to the (discrete) interference pattern observed in the double-slit experiment in the corresponding set-up (with both slits open and no devices installed allowing one to establish through which slit each quantum object passes). I am indebted to one of the article's reviewers for pointing out this aspect of the quantum-mechanical situation. However, these amplitudes (which are related to real measurable quantities) are not the same as the 'symbolic' amplitudes in question, which are complex quantities enabling us to predict the probabilities relating to the oscillations in question. This is why these amplitudes are seen as 'symbolic' by Bohr and Heisenberg, that is, as symbols borrowed from classical physics without having the physical meaning they have there. To cite Bohr:

> The symbolic character … of the artifices [of the quantum-mechanical formalism] also becomes apparent in that an exhaustive description of the electromagnetic wave fields leaves no room for light quanta and in that, in using the conception of matter waves, there is never any question of a complete description similar to that of the classical theories. Indeed, … the absolute value of the so-called phase of the waves never comes into consideration when interpreting the experimental results. In this connection, it should also be emphasized that the term 'probability amplitude' for the amplitude function of the matter waves is part of a mode of expression which, although often convenient, can, nevertheless, make no claim to possessing general validity [as concerns what is observed]. ([15], v. 1, p. 17)

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

His new theory offers the possibility of predicting, in general probabilistically, the outcomes of quantum experiments, but at the cost of abandoning a physical description of the ultimate objects considered, a cost unacceptable to some, even to most, beginning with Einstein. Heisenberg says: 'However, we shall see presently that also in quantum theory the phase has a definitive significance which is *analogous* to its significance in classical theory' ([32], p. 264; emphasis added). 'Analogous' could only mean here that, rather than being analogous physically, the way the phase enters mathematically is analogous to the way the classical phase enters mathematically in classical theory, in accordance with the *mathematical* form of the correspondence principle, insofar as quantum-mechanical equations are formally the same as those of classical physics. Heisenberg only considered a 'toy' model of a quantum aharmonic oscillator, and thus he needed only a Newtonian equation for it.

In this way, Heisenberg gave the correspondence principle a mathematical expression, even changed it into the mathematical correspondence principle. The variables to which these equations apply could not, however, be the same, because, if they were, the equations would not make correct predictions for low quantum numbers. As Heisenberg explains, if one considers

> a given quantity $x(t)$ [a coordinate as a function of time] in classical theory, this can be regarded as represented by a set of quantities of the form
>
> $$A_\alpha(n)\mathrm{e}^{iw(n)\alpha t},$$
>
> which, depending upon whether the motion is periodic or not, can be combined into a sum or integral which represents $x(t)$:
>
> $$x(n,t) = \sum_{+\infty}^{-\infty} {}_\alpha A_\alpha(n)\mathrm{e}^{i\omega(n)\alpha t}$$
>
> or
>
> $$x(n,t) = \int_{+\infty}^{-\infty} A_\alpha(n)\mathrm{e}^{i\omega(n)\alpha t}\, d\alpha. \quad ([32],\ \text{p. } 264)$$

Heisenberg next makes his most decisive and most extraordinary move. He notes that 'a similar combination of the corresponding quantum-theoretical quantities seems to be impossible in a unique manner and therefore not meaningful, in view of the equal weight of the variables n and $n-\alpha$' ([32], p. 264). 'However', he says, 'one might readily regard the ensemble of quantities $A(n, n-\alpha)e^{i\omega(n,n-\alpha)t}$ [an infinite square matrix] as a representation of the quantity $x(t)$' ([32], p. 264).

The arrangement of the data into square tables is a brilliant and, as I said, in retrospect, natural way to connect the relationships (transitions) between two stationary states. However, it does not by itself establish an *algebra* of these arrangements, for which one needs to find rigorous rules for adding and multiplying these elements—rules without which Heisenberg cannot use his new variables in the equations of the new mechanics. To produce a *quantum-theoretical interpretation* (which, again, abandons motion and other spatio-temporal concepts of classical physics at the quantum level) of the classical equation of motion that he considered, as applied to these new variables, Heisenberg needs to be able to construct the powers of such quantities, beginning with $x(t)^2$, which is actually all that he needs for his equation. The answer in classical theory is obvious and, for the reasons just explained, obviously unworkable in quantum theory. Now, 'in quantum theory', Heisenberg proposes,

> it seems that the simplest and most natural assumption would be to replace classical [Fourier] equations ... by
>
> $$B(n, n-\beta)\mathrm{e}^{i\omega(n,n-\beta)t} = \sum_{-\infty}^{+\infty} A(n, n-\alpha)A(n-\alpha, n-\beta)\mathrm{e}^{i\omega(n,n-\beta)t}$$

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

or

$$= \int_{-\infty}^{+\infty} A(n, n-\alpha) A(n-\alpha, n-\beta) \mathrm{e}^{i\omega(n,n-\beta)t}\, d\alpha. \quad \text{([32], p. 265)}$$

This is the main postulate, the (matrix) multiplication postulate, of Heisenberg's new theory, 'and in fact this type of combination is an almost necessary consequence of the frequency combination rules' ([32], p. 265). This combination of the particular arrangement of the data and the construction of an algebra of multiplying his new variables is Heisenberg's great invention. (As I noted, although some of them satisfy the canonical commutation relation, these matrices do not form an algebra with respect to the composition.) The 'naturalness' of this assumption should not hide the radical and innovative nature of this assumption or indeed discovery, one of the greatest in twentieth-century physics, even all physics.

Although it is commutative in the case of squaring a given variable, x^2, this multiplication is in general non-commutative, expressly for position and momentum variables, and Heisenberg, without quite realizing it, used this non-commutativity in solving his equation, as Dirac was the first to notice. Taking his inspiration from Einstein's 'new kinematics' of special relativity, Heisenberg spoke of his new algebra of matrices as the 'new kinematics'. This was not the best choice of term because his new variables no longer described or were even related to motion as the term kinematic would suggest, one of many historically understandable but potentially confusing terms. Planck's constant, h, which is a dimensional, dynamic entity, has played no role thus far. Technically, the theory was not even a mechanics, insofar as it did not offer a description of individual quantum processes, or of anything else. 'Observables', for the corresponding operators, and 'states', for Hilbert-space vectors, are other such terms: we never observe these 'observables' or 'states', but only use them to predict, probabilistically, what is observed in measuring instruments. To make these predictions, one will need Planck's constant, h, which thus enters as part of this new relation between the data in question and the mathematics of the theory.

Heisenberg's revolutionary thinking not only introduced a new type of mathematical model in physics but also established a new way of doing theoretical physics. Indeed, this way of thinking also redefined experimental physics. The practice of experimental physics no longer consists, as in classical physics, of tracking the independent behaviour of the systems considered. Instead, it consists of *unavoidably* creating configurations, by now almost unbelievable in their complexity (such as those found in the LHC), of experimental technology that reflect the fact that what happens is *unavoidably* defined by what experiments we perform, how we interact with quantum objects, rather than only by their independent behaviour. These configurations embody the effects of the interactions between quantum objects and measuring instruments, through which effects quantum objects are defined and, when possible, distinguished from one another, while always remaining beyond the reach of quantum theory and even thought itself. I emphasize 'unavoidably' because, while the behaviour of classical objects may be affected by experimental technology, in general, we can observe them without interfering with their behaviour, or can compensate for this interference so that we can describe this behaviour independently.

The practice of theoretical physics no longer consists of offering an idealized mathematical description of quantum objects and their behaviour. Instead, it consists of developing mathematical machinery that is able to predict, in general probabilistically or statistically, the effects in question, manifested as the outcomes of quantum events and of correlations between some of these events.

The situation takes a more radical form in QFT and experimental physics in the corresponding (high-energy) quantum regimes than in QM and experimental physics in the corresponding (low-energy) quantum regimes. While, in this type of interpretation, retaining the non-realist and non-causal epistemology of quantum phenomena and QM, the QFT situation is characterized by:

(1) more complex configurations of observed phenomena, defined by the effects of the interaction between quantum objects and measuring instruments and of configurations of

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

such instruments, in part reflecting the loss of the identity of elemental quantum objects even within a single experiment;

(2) a more complex structure of theoretical predictions and, hence, of the relationships between a possible mathematical formalism and the measuring instruments involved, and hence quantum objects and ultimately quantum fields, responsible for the effects observed in these instruments; and

(3) a more complex nature of the mathematical formalism and the mathematical procedures involved, responding to the situations described in (1) and (2).

The discovery of the Higgs boson may be the most spectacular recent example, followed more recently by the detection of a pentaquark. In this case, this technology was also supplemented by the, arguably, from now on irreducible, role of computer technology, which thus becomes a fundamental part of our modelling in physics. However, many other examples are found throughout the history of high-energy physics, beginning with the discovery of antimatter, a consequence of Dirac's equation and the inaugural event of this history. I shall now discuss Dirac's discovery of his equation for the relativistic electron, which, I argue, was the discovery of the mathematical technology of QFT. It is true that Dirac's 1928 theory of the electron was not even a fully fledged QED [36], let alone QFT, and it was not a field theory. Nevertheless, it introduced some of the most fundamental principles of QED and QFT.

4. 'The biggest of all the big changes': high-energy physics and the mathematical technology of quantum field theory

Three key elements were, in Dirac's view, required for a relativistic quantum equation for a free electron. The first, relativistic, element was that time and space must enter symmetrically, and be interchangeable, which was not the case in Schrödinger's equation, because it contains the first derivative of time and the second derivatives of coordinates. The second, quantum, element was that the equation had to contain the first-order derivative in time; this element is required by quantum-theoretical considerations, captured by the quantum-mechanical formalism, specifically by Schrödinger's equation and related to other key features of the formalism, especially the conservation of the probability current (which entails positive definite probability density). The third element was that the non-relativistic limit of a relativistic equation for the electron needed to be Schrödinger's equation. A proper equation for the electron thus needed to be a linear differential equation, first order in both space and time, because quantum theory requires the first-order derivative in time and relativity requires that space and time must enter symmetrically.

Although Dirac's thinking seems eminently reasonable in retrospect, it appears that only Dirac thought of the situation in this way at the time. His famous conversation with Bohr that occurred then is revealing [37]:

> Bohr: What are you working on?
>
> Dirac: I am trying to get a relativistic theory of the electron.
>
> Bohr: But Klein already solved that problem.

Dirac disagreed, and it is clear why he did, and why Bohr, who eventually came to greatly appreciate Dirac's theory, should have known better. The Klein–Gordon equation, to which Bohr referred, is relativistic and symmetrical in space and time, but it is not a first-order linear differential equation in either (both variables enter via the second derivatives). One can derive the continuity equation from it, but the probability density is not positive definite. By the same token, the Klein–Gordon equation does not give one Schrödinger's equation

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t)=\left[\frac{-\hbar^2}{2m}\nabla^2+V(\mathbf{r},t)\right]\psi(\mathbf{r},t),$$

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

in the non-relativistic limit, where m is the particle's mass, V is potential energy, ∇^2 is the Laplacian and ψ is the wave function (the position–space wave function). Schrödinger, who appears to have been the first to write down the Klein–Gordon equation in the process of his discovery of his wave mechanics, abandoned it in view of the incorrect predictions it gave in the non-relativistic limit. Dirac's equation does convert into Schrödinger's equation in the non-relativistic limit. At its immediate non-relativistic limit, Dirac's theory converts into Pauli's spin-matrix theory, while Schrödinger's equation, which does not contain spin, is the limit of Pauli's theory, if one neglects spin [38]. Dirac's equation enabled him to answer, for the relativistic electron, the question, 'What is the probability of *any dynamical variable* at any specified time having a value laying between any specified limits, when the system is represented by a given wave function ψ_n?', which the Klein–Gordon theory could only answer for 'the position of the electron ... but not [for] its momentum, or angular momentum, or any other dynamic variable' ([36], pp. 611–612; emphasis added). This is the main question of quantum theory, defined by the QP/QS principle, and, just as Heisenberg's scheme, that of Dirac could be viewed from the quantum-informational standpoint because it, too, described transmitting information, probabilistic or statistical in character, between measuring devices, but not how this information was transmitted by an electron.

In contrast with Heisenberg's scheme (which used the equations of classical mechanics), Dirac needed not only new variables but also a new equation. As in Heisenberg, Dirac's new variables were matrix-type variables, but of a more complex character, involving the so-called spinors and the multi-component wave functions. The latter is a crucial concept, discovered by Pauli in his non-relativistic theory of spin, but given a more rigorous foundation by Dirac. Dirac's spinors had never been used in physics previously, although they were introduced in mathematics by W. C. Clifford about 50 years earlier. Dirac was unaware of their existence and reinvented them in deriving his equation, similarly to the way Heisenberg reinvented matrix algebra in his discovery QM. Dirac's equation encodes a complex Hilbert-space machinery. Mathematically, the problem confronting Dirac may be seen as that of taking a square root of the Klein–Gordon equation, which implies that every solution of Dirac's equation is a solution of the Klein–Gordon equation, while the opposite is not true. The equation, as introduced by Dirac, was

$$\left(\beta mc^2 + \sum_{k=1}^{3} \alpha_k p_k c\right) \psi(x,t) = \mathrm{i}\hbar \frac{\partial \psi(x,t)}{\partial t}.$$

The new mathematical elements here are the 4×4 matrices α_k and β and the four-component wave function ψ. The Dirac matrices are all Hermitian,

$$\alpha_i^2 = \beta^2 = I_4$$

(I_4 is the identity matrix), and the mutually anti-commute:

$$\alpha_i\beta + \beta\alpha_i = 0$$

$$\text{and} \quad \alpha_i\alpha_j + \alpha_j\alpha_i = 0.$$

The above single symbolic equation unfolds into four coupled linear first-order partial differential equations for the four quantities that make up the wave function. The matrices form a basis of the corresponding Clifford algebra. One can think of Clifford algebras as quantizations of Grassmann's exterior algebras, in the same way that the Weyl algebra is a quantization of symmetric algebra. Here, $\boldsymbol{p}$ is the momentum operator in Schrödinger's sense, but in a more complicated Hilbert space than in QM. The wave function $\psi(t,\boldsymbol{x})$ takes the value in a Hilbert space $X = C^4$ (Dirac's spinors are elements of X). For each t, $\psi(t,\boldsymbol{x})$ is an element of $H = L^2(R^3; X) = L^2(R^3) \otimes X = L^2(R^3) \otimes C^4$.

Finding new matrix variables or Hilbert-space operators became the uniquely defining mathematical element of quantum theory. The current theories of weak forces, electroweak unifications and strong forces (QCD) were all discovered by finding such variables. This is correlative to establishing the transformation group, a Lie group, of the theory and finding

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

representations of this group in the corresponding Hilbert spaces. This is also true for Heisenberg's matrix variables, as was discovered by H. Weyl and E. Wigner, for Heisenberg's group. In elementary-particle theory, irreducible representations of such groups correspond to elementary particles, an idea that was one of Wigner's major contributions [39]. This was, for example, how M. Gell-Mann discovered quarks, because at the time there were no particles corresponding to the irreducible representations (initially there were three of those, corresponding to three quarks) of the symmetry group of the theory, SO(3). It is the group of all rotations around the origin in three-dimensional space, R^3, rotations represented by all three by three orthogonal matrices with determinant 1. (This group is non-commutative.) The electroweak group that Gell-Mann helped to find is SU(2), the group of two by two matrices with the determinant 1. Quarks are part of both theories. As we have seen, these symmetries and also U(1) (which combined with SU(2) defines the electroweak unification) figure in and imply the Higgs mechanism.

Before proceeding to his derivation of his equation, Dirac comments on another 'difficulty' of the Klein–Gordon equation:

> [The equation] refers equally well to an electron with charge e as to one with charge −e. If one considers for definitiveness the limiting case of large quantum numbers one would find that some of the solutions of the wave equation are wave packets moving in the way a particle of −e would move on the classical theory, while others are wave packets moving in the way a particle with charge e would move classically. For this second class of solutions W has a negative value. One gets over the difficulty on the classical theory by arbitrarily excluding those solutions that have a negative W. One cannot do this on the quantum theory, since in general a perturbation will cause transitions from states with W positive to states with W negative. Such a transition would appear experimentally as the electron suddenly changes its charge from −e to e, a phenomenon which has not been observed. The true relativi[stic] wave equation should thus be such that its solutions split up into two non-combining sets, referring respectively to the charge −e and the charge e. In the present paper we shall only be concerned with the removal of the first of these difficulties. The resulting theory is therefore still only an approximation, but it appears to be good enough to account for all the duplexity phenomena without arbitrary assumptions. ([36], p. 612)

Dirac's theory inherits this problem of the Klein–Gordon theory, because mathematically every solution of Dirac's equation is a solution of the Klein–Gordon equation, of which Dirac's equation is a square root (the opposite is not true). Heisenberg's assessment was even stronger:

> The classical theory could eliminate this difficulty by arbitrarily excluding the one sign, but this is not possible to do according to the principles of quantum theory. Here spontaneous transitions may occur to the states of negative value of energy E; as these have never been observed, *the theory is certainly wrong*. Under these conditions it is very remarkable that the positive energy-levels (at least in the case of one electron) coincide with those actually observed. ([4], p. 102)

'The theory is certainly wrong'—no less!

Dirac's theory, famously, proved to be better than it appeared at the time even to its creator. That, in general, a perturbation will cause transitions from states with positive E to states with negative E, and that such a transition would appear experimentally as the electron suddenly changes its charge from $-e$ to e, is what actually happens, and it was experimentally established in a year or so. It took a few years to realize that it is antimatter and that this type of transition defines high-energy regimes in quantum physics. Dirac's theory amounted to the introduction of mathematical formalism (in its essential features that of QFT) that is able to respond to the richer, more multiple configuration of effects observed in QFT regimes. Dirac aimed at a relativistic equation for an electron, as a relativistic extension of QM, but he arrived at something else, or rather the relativistic behaviour of the electron has proved to be something entirely different from what was expected even after QM. Dirac's equation unexpectedly captured this different

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

behaviour. The equation was, as I said, not written for a quantum field (it was a particle equation) nor was it sufficient for an adequate treatment of many QED phenomena, such as those that resulted from scattering processes, which require a fully fledged QED formalism. However, the key ingredients of this new physics were established or implied by his equation.

Dirac's derivation of his equation is a remarkable example of both Dirac's highly original way of thinking and the new practice of theoretical physics, introduced by Heisenberg. As explained above, in this way of doing theoretical physics, one no longer aims at offering an idealized mathematical representation of quantum objects and their behaviour, but at developing mathematical technology that is able to predict, probabilistically or statistically, the effects of the interactions between quantum objects and measuring instruments. I shall, however, bypass Dirac's derivation of his equation, which I have considered from this perspective in [40]. It is this paradigm that is my main interest here.

Dirac's equation, as I said, encodes a complex mathematical architecture. The Hilbert space associated with given quantum systems in Dirac's theory is a tensor product of the infinite-dimensional Hilbert space (encoding the mathematics of continuous variables) and a finite-dimensional Hilbert space over complex numbers, which, in contrast with the two-dimensional Hilbert space of Pauli's theory, C^2, is four-dimensional in Dirac's theory, C^4. (Spin is contained by the theory automatically.) Dirac's wave function $\psi(t, \boldsymbol{x})$ takes the value in a Hilbert space $X = C^4$ (Dirac's spinors are elements of X). For each t, $\psi(t, \boldsymbol{x})$ is an element of

$$H = L^2(R^3; X) = L^2(R^3) \otimes X = L^2(R^3) \otimes C^4.$$

Other forms of QFT give this type of architecture an even greater complexity, responding to the complexity of experimental situations, defined by the interactions between quantum objects or fields and measuring instruments, in the corresponding high-energy regimes. All phenomena considered are discrete, and QFT provides only probabilities or statistics for the outcomes of quantum events registered in measuring instruments. The architecture of QFT mathematically responds to, and helped the discovery of, the following physical situation.

Suppose that one arranges for an emission of an electron, at a given high energy, from a source and then performs a measurement at a certain distance from that source. Placing a photographic plate at this point would do. The probability of the outcome would be properly predicted by QED. But what will be the outcome? The answer is not what our classical or even our quantum-mechanical intuition would expect, and this unexpected answer was a revolutionary discovery by Dirac. Once the process occurs at a high energy and is governed by QED, beginning with Dirac's equation, the situation is radically different from that in QM. One might find, in the corresponding region, not only an electron or nothing, as in quantum-mechanical regimes, but also other particles: a positron, a photon, an electron–positron pair, etc. QED correctly predicts which among such events can occur, and with what probability, and, in the present view, it can only predict such probabilities, or statistical correlations between certain quantum events. In order to do so, the corresponding Hilbert-space machinery becomes much more complex, essentially making the wave function ψ a four-component Hilbert-space vector, as opposed to a one-component Hilbert-space vector, as in QM. This Hilbert space is, as noted, $H = L^2(R^3; X) = L^2(R^3) \otimes X = L^2(R^3) \otimes C^4$ in the case of a free electron, governed by Dirac's equation.

Once we move to still higher energies or those governed by the corresponding forms of QFT the panoply of possible outcomes becomes much greater. The Hilbert spaces involved would be given a yet more complex structure, in relation to the appropriate Lie groups and their representations, defining (when these representations are irreducible) different elementary particles. In the case of QED we only have electrons, positrons and photons. It follows that in QFT an investigation of a particular type of quantum object involves not only other particles of the same type but also other types of particles. The identity of particles within each type is strictly maintained in QFT, as it is in QM. In either theory one cannot distinguish different particles of the same type, such as electrons. One can never be certain that one encounters the same electron in the experiment just described even in the quantum-mechanical situation, although the probability

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

that it would be a different electron is low in the QM regime in comparison with that in the regime of QED. In QFT, instead of identifiable moving objects and motions of the type studied in classical physics, we encounter a continuous emergence and disappearance, creation and annihilation, of particles from point to point, theoretically governed by the concept of virtual particle formation. The operators used to predict the probability of such events are the creation and annihilation operators. QM still preserves the identity of quantum objects, through a certain set of effects in measuring instruments (the only way in which quantum objects, such as elementary particles, manifest their existence, at least in non-realist interpretations). In QFT, this identity is no longer preserved even in a single experiment.

The introduction of this new mathematical formalism was a momentous event in the history of quantum physics, comparable to that of Heisenberg's introduction of matrix variables. Heisenberg saw Dirac's theory as

> perhaps the biggest change of all the big changes in physics of our century. It was a discovery of utmost importance because it changed our whole picture of matter. … It was one of the most spectacular consequences of Dirac's discovery that the old concept of the elementary particle collapsed completely. ([41], pp. 31–33)

The volume, written in the 1970s, contains a companion article 'What is an elementary particle?' This question is still unanswered. Heisenberg elaborated:

> So the final result at this point seems to be that Dirac's theory of the electron has changed the whole picture of atomic physics. After abandoning the old concept of the elementary particles, those objects which had been called 'elementary particles' have now to be considered as complicated compound systems, and will have to be calculated some day from the underlying natural law, in the same way as the stationary states of complicated molecules will have to be calculated from quantum or wave mechanics. We have learned that energy becomes matter when it takes the form of elementary particles. The states called elementary particles are just as complicated as the states of atoms and molecules. Or, to formulate it paradoxically: every particle consists of all other particles. Therefore we cannot hope that elementary particle physics will ever be simpler than quantum chemistry. This is an important point, because even now many physicists hope that some day we might discover a very simple route to elementary particle physics, as the hydrogen spectrum was in the old days. This, I think, is not possible. ([41], pp. 34–35)

Unanswered as it remains, the question 'What is an elementary particle?' or (they are fundamentally related or are even part of the same question) 'What is a quantum field?' was advanced immeasurably by Dirac's equation and then QED and QFT.[12] The theory made remarkable progress since these remarks by Heisenberg, as is manifest in the electroweak unification and the quark–gluon model of nuclear forces, both underlined by the Higgs field, developments that commenced around the time of these remarks.[13] Some of them, at their earlier stage, were noted in Heisenberg's articles here cited, which also emphasized the role of symmetry and group theory, central to these developments. Many predictions of the theory, from quarks to electroweak bosons and the concepts of confinement and asymptotic freedom, were spectacular. It was also QFT that led to string and then brane theories. However, the essential epistemological points and principles here considered have remained in place, just as they have in QM. In this respect, Heisenberg's assessment just cited would require relatively minor adjustments. A more recent statement by A. Pais, who was both a major practitioner of the theory and a major historian of the subject, confirms this. The statement is not that recent either. It was made in 1986, but not much has changed in this respect since, as is clear, for example, from F. Wilczek's 2005 review of

[12]The title was reprised by S. Weinberg's 1996 article, reflecting on a more advanced stage of QFT, without, however, answering the question either [42].

[13]For an introduction to the current state of QFT, see [43] and references therein, which include standard treatments of the subject, such as, to give representative physical, philosophical and historical examples, [44–47] and a few updates.

rsta.royalsocietypublishing.org Phil. Trans. R. Soc. A 374: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

the present state of the theory [48] or from more recent assessments, although QED continues to add new technical findings. According to Pais:

> Is there a theoretical framework for describing how particles are made and how they vanish? There is: quantum field theory. It is a language, a technique, for calculating the probabilities of creation, annihilation, scattering of all sorts of particles: photons, electrons, positrons, protons, mesons, others, by methods which to date invariably have the characters of successful approximations. No rigorous expression for the probability of any of the above-mentioned processes has ever been obtained. The same is true for the corrections, demanded by quantum field theory, for the positions of energy levels of bound-state systems [e.g. atoms]. There is still a [Schrödinger] equation for the hydrogen atom, but it is no longer exactly soluble in quantum field theory. In fact, in a sense to be described [i.e. the sense explained above], the hydrogen atom can no longer be considered to consist of just one proton [or three quarks plus gluons in the nucleus] and one electron. Rather it contains infinitely many particles. ([49], p. 325)

Pais's added comment is also still valid:

> In quantum field theory the postulates of special relativity and of quantum mechanics are taken over unaltered, and brought to a synthesis which perhaps is not yet perfect but which indubitably constitutes a definitive step forward. It is also a theory which so far has not yielded to attempts at unifying the axioms of general relativity with those of quantum mechanics. Is quantum field theory the ultimate framework for understanding the structure of matter and the description of elementary processes? Perhaps, perhaps not. ([49], p. 325)

Heisenberg's statement cited above gives the situation another philosophical dimension by suggesting that Dirac's discovery revealed that in fundamental physics, at least if it is considered quantum, the complex *preceded* the simple (meaning this logically, rather than ontologically), which makes all simplicity provisional. We cannot hope to decompose the complex into essentially, fundamentally more simple constituents, but only into structurally equal or even more complex ones, which is not to say that such a decomposition is not necessary or important. We still speak of elementary particles and make good use of this concept.

All this does not mean that QFT is not in need of further advancement or does not have problems. It is, as Pais said, 'not yet perfect', and may not even ever become perfect. Some of these problems may and eventually must lead to developments that will be beyond the power of QFT, and will require replacing it by a new theory. Mathematically, such a new theory might, as QFT was with respect to QM, be a more natural outgrowth of QFT, or it might be something entirely different. It may be more classical in nature at the next stage of the history of physics (the only form of finality there is), thus fulfilling Einstein's hope, shared by many. Conversely, it may be something even more radical than anything we have seen or can imagine, just as happened with relativity and QM at the time of their emergence.

Renormalization may be seen as a problem, even with the effective theories and renormalization group approaches in mind, which suggests that, once we reach a sufficiently deep theory (as opposed to our current approximations), it will be finite. Perhaps! One could hardly be certain. In the first place, such a theory remains a tall order, even if one leaves quantum gravity aside, and one might not be able to leave it aside in such as theory. All such theories remain, however, hypothetical thus far, with very uncertain futures. But then, how bad is renormalization? It has worked extremely well thus far, at least, again, insofar as our quantum theories can disregard gravity. Our future fundamental theories might prove to be finite (some versions of string and brane theory appear to hold such a promise), thus proving that the necessity of renormalization is merely the result of the limited reach of our quantum theories at present. (Effective quantum field theories are based on this view.) The emergence of other finite alternatives, proceeding along entirely different trajectories, may not be excluded either. While, however, a finite theory may be preferable, renormalization may not be a very big price

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

to pay for the theory's extraordinary capacity to predict the increasingly complex manifold of quantum phenomena that physics has confronted throughout the history of QFT. And then, a finite theory may not be possible, in which case renormalization will continue to be our main hope and instrument. QFT continues to provide an extraordinarily effective mathematical technology for handling new phenomena observed in new configurations of measuring instruments. The LHC is the latest example of the machine (in either sense) for creating such configurations, one of which (a very complex one) enabled us to confirm the existence of the Higgs boson. Of course, as all technologies, the mathematical technology of QFT and the experimental technology of the type of the LHC might, and even one day must, become obsolete and be superseded by other technologies. Thus far, however, both have worked marvellously, beginning with Dirac's introduction of QED and his equation for the relativistic electron, which remains a spectacular and illuminating case, leading to the discovery of the Higgs boson. A pentaquark is the latest discovery, but undoubtedly not the last. New discoveries are bound to follow and may confirm the power of QFT or defeat it, and some very recently observed, but not yet confirmed, event suggests tantalizing new possibilities. A defeat could be more exciting, because it will require a new theory, perhaps an entirely new kind of theory. Either way physics will win.

5. Circuits and mathematical structures: on quantum information theory

I now turn to quantum information theory, using as my starting point the works of M. G. D'Ariano and co-workers, G. Chiribella and P. Perinotti, which also led them to the derivation of Dirac's equation from the principle of quantum information theory alone, without using the principle of relativity [34]. For the moment, I shall consider the authors' programme of QFDT based in these principles. The programme is inspired by 'a need for a *deeper understanding of quantum theory* itself from fundamental principles', which, the authors contend, has never been really achieved, and is motivated by the development of quantum information theory ([50], p. 1). Among the key predecessors are C. A. Fuchs (e.g. [51]) and L. Hardy [52] who were equally motivated by the aim of deriving QM from a more natural set of principles, postulates or axioms.[14] Hardy's paper was, arguably, the first derivation of that type. Neither of these approaches is realist, nor, again, is that of D'Ariano and co-workers, or in any event they allow for non-realist interpretations. All these attempts are thus far limited to QFTD.[15]

According to Chiribella, D'Ariano and Perinotti:

> [T]he rise of quantum information science moved the emphasis from logics to information processing. The new field clearly showed that the mathematical principles of quantum theory imply an enormous amount of information-theoretic consequences. . . . The natural question is whether the implication can be reversed: is it possible to retrieve quantum theory from a set of purely informational principles? ([50], p. 1)

They aim to offer 'a complete derivation of QFDT based on purely operational informational principles':

> In this paper we provide a complete derivation of finite dimensional quantum theory based on purely operational principles. Our principles do not refer to abstract properties of the mathematical structures that we use to represent states, transformations, or measurements, but only to the way in which states, transformations, and measurements combine with each other. More specifically, our principles are of *informational* nature: they assert basic

[14]See [50] for further references. For comprehensive introductions to the subject of quantum information theory, see [53] and [54]. For a visionary introduction to the grounding ideas of quantum information theory, see J. A. Wheeler's manifesto [55]. Fuchs's work more recently 'mutated' to a somewhat different programme, that of quantum Bayesianism or QBism (e.g. [56]).

[15]While stressing the key fundamental principles that helped to establish these theories, this article does not claim that a sufficient derivation of either theory from such principles has been achieved. This remains an open question, even more so when dealing with continuous variables (to which most of my discussion has been restricted thus far), where the application of the principles of quantum information theory is also more complex.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

> properties of information processing, such as the possibility or impossibility to carry out certain tasks by manipulating physical systems. In this approach the rules by which information can be processed determine the physical theory, in accordance with Wheeler's program 'it from bit,' for which he argued that 'all things physical are information-theoretic in origin' [55]. Note [however, that] our axiomatization of quantum theory is relevant, as a rigorous result, also for those who do not share Wheeler's ideas on the informational origin of physics. In particular, in the process of deriving quantum theory we provide alternative proofs for many key features of the Hilbert-space formalism, such as the spectral decomposition of self-adjoint operators or the existence of projections. The interesting feature of these proofs is that they are obtained by manipulation of the principles, without assuming Hilbert spaces from the start. ([50], p. 1)[16]

Among these principles, the purification principle plays a particularly, even uniquely, important role, as an essentially quantum principle:

> The main message of our work is simple: within a standard class of theories of information processing, quantum theory is uniquely identified by a single postulate: *purification*. The purification postulate, introduced in [57], expresses a distinctive feature of quantum theory, namely that the ignorance about a part is always compatible with the maximal knowledge of the whole. The key role of this feature was noticed already in 1935 by Schrödinger in his discussion about entanglement ([58], p. 555), of which he famously wrote 'I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.' In a sense, our work can be viewed as the concrete realization of Schrödinger's claim: the fact that every physical state can be viewed as the marginal of some pure state of a compound system is indeed the key to single out quantum theory within a standard set of possible theories. It is worth stressing, however, that the purification principle assumed in this paper includes a requirement that was not explicitly mentioned in Schrödinger's discussion: if two pure states of a composite system AB have the same marginal on system A, then they are connected by some reversible transformation on system B. In other words, we assume that all purifications of a given mixed state are equivalent under local reversible operations. ([50], p. 2)

Chiribella, D'Ariano and Perinotti also speak of 'the purification postulate', and they refer to the remaining informational 'postulates' as 'axioms', because 'as opposed to the purification "postulate", … they are not at all specific [to] quantum theory' ([50], p. 3). They defined their operational principle qua principle later ([50], p. 6). While these terminological distinctions may be somewhat tenuous, they do not affect Chiribella, D'Ariano and Perinotti's main argument.

The purification principle is a new principle. Beyond the fact that it has a richer content than does Schrödinger's statement, Schrödinger never saw his claim as a principle, perhaps because of his critical view of QM. The principle could be related to Bohr's complementarity, which implies and even states that 'ignorance about a part [one of the two complementary parts] is always compatible with the maximal knowledge of the whole', making the very language of 'parts' and 'whole' provisional. Chiribella, D'Ariano and Perinotti's formulation of the purification principle to the effect that 'every physical state can be viewed as the marginal of some pure state of a compound system' avoids this difficulty. Bohr saw the EPR experiment (the background for Schrödinger's claim and his concept of entanglement) as a manifestation of complementarity ([15], v. 2, p. 59, [59]). Thinking in terms of 'circuits' (not found in Schrödinger) is close to Bohr's view of the role of measuring instruments in the constitution of quantum phenomena. One must keep in mind that all knowledge in question is probabilistic or statistical, defined by what Schrödinger calls 'expectation-catalogs', say, those provided by the wave function ([16], p. 158).

While having an essential and even unique role in Chiribella, D'Ariano and Perinotti's operational derivation of the QFDT, the purification principle is not sufficient to do so. Chiribella, D'Ariano and Perinotti need five additional axioms. This is not surprising. As discussed above,

[16]Note that Heisenberg and Dirac did not assume Hilbert spaces from the start either, but rather *arrived* at them, even if not quite *derived* them, from the fundamental principles they assumed.

rsta.royalsocietypublishing.org Phil. Trans. R. Soc. A 374: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

in Heisenberg's work, the main grounding quantum physical principles—the suspension of the representation of quantum objects and processes and the quantum probability and statistics (QP/QS) principle—were not sufficient to derive QM. To do so, he needed the correspondence principle, to which he gave a mathematical expression. Dirac, too, needed a larger set of principles, postulates and assumptions (even apart from those of relativity) for deriving his equation. What is remarkable, however, is that one needs only one 'postulate' to distinguish classical and quantum statistical-information theories. A similar situation transpires in Hardy's paper mentioned above, where this difference turns not only on a single 'axiom', but on the use of a single word, 'continuity', technically a single feature of the situation, 'the *continuity* of a reversible transformation between any two pure states' ([52], p. 2; emphasis added). Hardy's continuity axiom is different from Chiribella, D'Ariano and Perinotti's purification postulate. The point is that in both cases one axiom is sufficient to differentiate classical from quantum systems. On the one hand, there may not be a single system of axioms or postulates from which QFDT could be derived. On the other hand, certain characteristic traits, *the* characteristic traits, beginning with the role of h, may reflect something singular about quantum phenomena. This is more immediately apparent in the case of Schrödinger's 'characteristic trait' or (correlatively) Chiribella, D'Ariano and Perinotti's postulate, which deal with entanglement. But it is equally true in the case of Hardy's continuity axiom. According to M. Dickson:

> The quantum state is 'continuous' (in Hardy's sense) because for any two pure states, there is another pure state that is 'between' them, and in fact this 'middle' state is a superposition of the two original states. In other words, continuity holds precisely because the superposition *principle* holds. Continuity fails in the classical theory because the superposition principle fails there. From this point of view, it is less surprising—though not necessarily less important—that continuity is what makes the difference, in Hardy's framework, between classical and quantum theory. ([60], p. 321; emphasis added)

The superposition principle, I might add, is connected to both complex numbers and non-commutativity in the formalism, and, via Born's or related rules, to the probabilistic or statistical nature of the theory, in accordance with the QP/QS principle, embedded in both Chiribella, D'Ariano and Perinotti's and Hardy's axioms.

There are instructive specific parallels between Chiribella, D'Ariano and Perinotti's and Heisenberg's approaches. The QP/QS principle is present in both cases, given that Chiribella, D'Ariano and Perinotti see QFDT as an 'operational–probabilistic theory' of a special type, defined by the purification postulate ([50], p. 3). As they say:

> The operational-probabilistic framework combines the operational language of circuits with the toolbox of probability theory: on the one hand experiments are described by circuits resulting from the connection of physical devices, on the other hand each device in the circuit can have classical outcomes and the theory provides the probability distribution of outcomes when the devices are connected to form closed circuits (that is, circuits that start with a preparation and end with a measurement). ([50], p. 3)

This is similar to Heisenberg's thinking, as discussed here, although the concept of 'circuit', not found in Heisenberg, is closer to Bohr's view of the role of measuring apparatus in his interpretation. Heisenberg found his formalism of QM by using the mathematical correspondence principle, which allowed him to adopt the equations of classical mechanics, while changing the classical variables. This was not exactly the first principle, because it depended on the equations of classical mechanics, which one might prefer to be a consequence of fundamental quantum principles. On the other hand, the principle was related to the fact that the observable parts of measuring instruments are classically describable 'circuits', just as they are in Chiribella, D'Ariano and Perinotti's framework. Heisenberg needed new variables because the classical variables did not give Bohr's frequency rules for spectra, and the corresponding probabilities or statistics. Heisenberg discovered, almost by a guess, that these rules are satisfied by, in

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

general, non-commuting matrix variables with complex coefficients, related to amplitudes, from which one derives, by means of a Born-type rule, probabilities or statistics for transitions between stationary states, transitions manifested in spectra, observed in measuring devices. In this way, Heisenberg's derivation was essentially related to the fact that the observable parts of measuring instruments are classically describable devices, which are akin to 'operational circuits' in Chiribella, D'Ariano and Perinotti's or related quantum-informational schemes, such as that of Hardy, except that these schemes, in order to derive QFDT, start with giving a *mathematical structure* to these circuits themselves.

This approach is motivated by Chiribella, D'Ariano and Perinotti's aim to arrive at the mathematical architecture of QFDT in a more first-principle-like way; in particular, independently of the mathematics of classical physics. (The latter, to begin with, does not have discrete variables, such as spin, which are purely quantum, with which QFDT is associated.) The rules governing the structure of operational circuits are indeed *more empirical*. These rules, however, are not completely empirical, because (beyond the fact that no rules could even be completely empirical) operational circuits are given a mathematical structure. This is *parallel* to Heisenberg's arrangement of the quantities he used into the square tables, matrices, which we now take for granted, but which, as explained earlier, was a mathematical invention that gave a structure, architecture to these quantities, ultimately linked to the probabilities of transitions between stationary states. In fact, these quantities could even be seen in relation to circuits where these quantities are observed or (probabilistically) predicted as spectra. Heisenberg, again, needs other elements, just described, to establish this architecture fully, quite differently from how Chiribella, D'Ariano and Perinotti aim to do this. Nevertheless, in this case, too, we still deal with the architecture, that of circuits, from which the mathematical architecture of the theory emerges.[17] We keep in mind that in both approaches we only deal with mathematical structures linked to and providing the probabilities or statistics of the outcomes of discrete quantum experiments, thus in accord with the QD and QP/QS principles, without providing any representation of quantum processes themselves. As Chiribella, D'Ariano and Perinotti say: 'The rules summarized in this section define the operational language of circuits, which has been discussed in detail in a series of inspiring works by Coecke' ([50], p. 3). Coecke and others working in this area primarily aimed at recasting the architecture of at least the finite-dimensional Hilbert-space formalism into a new language, rather than deriving QFTD. According to Coecke:

> The underlying mathematical foundation of this high-level diagrammatic formalism relies on so-called *monoidal categories*, a product of a fairly recent development in mathematics. Its logical underpinning is *linear logic*, an even more recent product of research in logic and computer science. These monoidal categories do not only provide a natural foundation for physical theories, but also for proof theory, logic, programming languages, biology, cooking. ... So the challenge is to discover the necessary additional pieces of structure that allow us to predict genuine quantum phenomena. These additional pieces of structure represent the capabilities nature has provided us with to manipulate entities subject to the laws of quantum theory. ([64], p. 1)

The concept of circuit, thus, represents the arrangements of measuring instruments that are capable (only certain instruments are) of quantum measurements and predictions, which are probabilistic or statistical, and sometimes, as with the EPR or the EPR–Bohm type of experiments, are correlated, and as such have specific architecture. This is also a realist representation of these arrangements, which is possible, because they are described by classical physics, even though they interact with quantum objects, and thus have a quantum stratum, which enables this interaction, disregarded by this classical representation without any detriment to either

[17] A remarkable precursor to this approach is Schwinger's framework of 'the algebra [of the symbols] of quantum measurements', which in effect extends from Heisenberg's approach and Bohr's thinking, from which Schwinger borrows the terms 'kinematic' and 'symbolic', respectively [61,62]. For a discussion of Schwinger's framework, see [63].

rsta.royalsocietypublishing.org *Phil. Trans. R. Soc. A* **374**: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

measurements or predictions.[18] The specific architecture and the properties of circuits, '*the necessary additional pieces of structure*', that help and ideally enable us to derive the formalism of QFDT, or one might say of the arrangements of elements (such as Hilbert-space operators) that define this formalism. This formalism, again, has probabilistically or statistically predictive relations to what is observed, without, in non-realist interpretations, representing quantum objects and processes.

One can gain further insights into the architecture of circuits from Hardy's approach, where circuits play an equally central role. He arrives at a different set of main assumptions necessary to derive QFDT from those of Chiribella, D'Ariano and Perinotti, but the main strategy is the same: it is to establish the architecture of circuits that, perhaps with additional axioms, allows one to derive the mathematical formalism of QFDT. According to Hardy:

> Circuits have:
>
> — A setting, s(H), given by specifying the setting on each operation.
> — An outcome set, o(H), given by specifying the outcome set at each operation (equals o(A) × o(B) × o(C) × o(D) × oE in this case). We say the fragment 'happened' if the outcome is in the outcome set.
> — A wiring, w(E), given by specifying which input/output pairs are wired together. ([65], p. 7)

With this definition in hand, I shall consider some of Hardy's fundamental assumptions in a different paper, which make my main point here more transparent:

> We will make two assumptions to set up the framework in this paper. …
>
> **Assumption 1.** The probability, Prob (A), for any circuit, A (this has no open inputs or outputs), is well conditioned. It is therefore determined by the operations and the wiring of the circuit alone and is independent of settings and outcomes elsewhere. ([66], p. 11)

This is a physical postulate, essentially that of spatial and temporal locality (or the corresponding causality) combined with probability or statistics, along the lines of the QP/QS principle. The task now becomes how to derive a QFDT that could correctly predict these probabilities. One needs another assumption:

> **Assumption 2.** Operations are fully decomposable. … We assume that any operation $\mathrm{A}_{a1b2\ldots c3}^{\mathrm{d4e5\ldots f6}}$ can be written, … in a symbolic notation,
>
> $$\mathrm{A}_{a1b2\ldots c3}^{\mathrm{d4e5\ldots f6}} \equiv^{\mathrm{d4e5\ldots f6}} \mathrm{A}_{a1b2\ldots c3} \mathrm{X}_{\mathrm{a1}}^{a1} \mathrm{X}_{\mathrm{b2}}^{b2} \cdots \mathrm{X}_{\mathrm{c3}}^{c3}\, {}_{d4}\mathrm{X}^{\mathrm{d4}}\, {}_{e5}\mathrm{X}^{\mathrm{e5}} \cdots {}_{f6}\mathrm{X}^{\mathrm{f6}}$$
>
> In words we will say that any operation is equivalent to a linear combination of operations each of which consists of an effect for each input and a preparation for each output. …
> We allow the possibility that the entries in ${}^{\mathrm{d4e5\ldots f6}}\mathrm{A}_{\mathrm{a1b2\ldots c3}}$ are negative (and this will, indeed, be the case in quantum theory). Hence, in general, this cannot be thought of as physical mixing. …
>
> Assumption 2 introduces a *subtly* different attitude than the usual one concerning how we think about what an operation is. Usually we think of operations as effecting a transformation on systems as they pass through. Here we think of an operation as corresponding to a bunch of separate effects and preparations. We need not think of systems as things that preserve their identity as they pass through—we do not use the same labels for wires coming out as going in. This is certainly a more *natural* attitude when there can be different numbers of input and output systems and when they can be of different types. Both classical and quantum transformations satisfy this assumption. In spite of the different attitude just mentioned, we can implement arbitrary transformations, such as

[18]The information thus obtained is also physically classical but its architecture and mode of transmission is quantum, that is, it cannot be classically generated.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

> unitary transformations in quantum theory, by taking an appropriate sum over such effect and preparation operations. ([66], pp. 19–20; emphasis added)

After a technical discussion of 'duotensors' (which I put aside here), Hardy suggests a principle:

> *Physics to mathematics correspondence principle*. For any physical theory, there [exists] a small number of simple hybrid statements that enable us to translate from the physical description to the corresponding mathematical calculation such that the mathematical calculation (in appropriate notation) looks the same as the physical description (in appropriate notation).
>
> Such a principle might be useful in obtaining new physical theories (such as a theory of quantum gravity). Related ideas to this have been considered by category theorists [67]. A category of physical processes can be defined corresponding to the physical description. A category corresponding to the mathematical calculation can also be given. The mapping from the first category to the second is given by a functor (this takes us from one category to another). ([66], p. 39)

This 'hybrid' construction is necessary to give structure or architecture to circuits, so that it could then be translated into a proper formalism of QFDT, thus establishing QFDT itself as a proper QT. In this case the translation, the 'functor' in question, is 'virtually direct'. As noted, Coecke and other category theorists establish this type of 'functor' by recasting the Hilbert-space formalism of QFDT so as to connect the two categories in question—that of circuits and that of mathematical structures, enabling proper predictions—rather than derive this functor from axioms or postulates, as Hardy or D'Ariano and co-workers aim to do.

Circuits, then, and their arrangements symbolically embody or represent the architecture of measuring instruments capable of detecting and measuring quantum events, and also enabling the probabilistic predictions of future events. Their arrangement and operations in the second respect is enabled by rules that should ideally be derived from certain sufficiently natural assumptions, independent, for example, of the correspondence principle, which make quantum theory depend on classical physics. On the other hand, circuits and their arrangements are still described classically, a fact related to Bohr's correspondence principle, a connection that, however, requires a separate analysis.

An important difficult question is the relationships between the architecture of the 'circuits', that is, experimental arrangements used in the experiments, in the standard (infinite-dimensional) QM, and the architecture of the theory, or the same relationships in QFT. Consider the double-slit experiment, say, in the interference pattern set-up (both slits are open and no counters installed allowing one to determine through which slit each particle passes). It is a circuit, which embodies (or could be represented by a scheme that embodies) preparations, measurements and predictions, the latter manifested in the interference pattern. However, then mathematical architecture enabling these predictions is a combination of the formal architecture of the equations of QM, equations taken from classical physics by the mathematical correspondence principle, and new types of variables, 'guessed' by Heisenberg, rather than derived from the architecture of circuits used in the experiments in question in QM.[19] I would not presume to rigorously describe the set-up of the double-slit experiment as a circuit, but it is one nevertheless, a complex one, albeit child's play in comparison with the circuitry found in high-energy quantum physics, such as that of the LHC.

Such questions will, however, need be addressed if one is to extend these types of derivation of QFDT to QM or to QED and QFT, or beyond as both Hardy and D'Ariano and co-workers aspire to do, along different lines. For, important as this task may be, it is hardly sufficient for these programmes to merely derive certain already established theories. Their primary value ultimately lies in solving still outstanding problems, in what they can do for the future of fundamental physics. Hardy aims to rethink general relativity in operational terms, analogous to those of

[19] Cf. Hardy's comment in ([52], p. 26).

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

QFDT, and to reach, in principle, quantum gravity by combining the operational frameworks of quantum theory (including QFT) and general relativity (e.g. [68]). D'Ariano and co-workers, by contrast, move from QFDT to QFT, via the concept of quantum cellular automata (which makes their framework no longer strictly operational), again with the ultimate goal of quantum gravity, but possibly bypassing general relativity as an intermediate stage. Their derivation of Dirac's equation is a major step in this direction, and, unlike Dirac's own, it only uses (along with other quantum-informational principles) the principle of locality, rather than those of special relativity [34]. I have discussed this derivation elsewhere [40]. I leave the subject by venturing a surmise that the principles of quantum information theory may play a major role in the future of fundamental physics, as, as I have argued, they, implicitly, have had in quantum theory, beginning with Heisenberg's work. As such, they can lead to alternative trajectories to those currently dominant in fundamental physics, say, those aiming to develop quantum gravity.

6. 'The question concerning technology' in quantum physics

I began my discussion of QM with Bohr's comment on Heisenberg's matrix mechanics in the last section, 'The development of a rational quantum mechanics', of 'Atomic theory and mechanics' ([15], v. 1, pp. 25–51), written in the wake of Heisenberg's and Born and Jordan's work, but before Schrödinger's introduction of wave mechanics. Bohr stressed that matrix mechanics was a strictly probabilistically or statistically predictive theory that did not offer a representation of quantum processes, even (in contrast with classical mechanics or relativity) individual ones, in space and time. In other words, it was, or was interpreted by Bohr, as well as its creators, as non-realist. I close by citing Bohr's final statement there on the new type of relationships between mathematics and mechanics in QM in a non-realist interpretation, stemming from this view. This statement might be unexpected given Bohr's customary insistence on the defining role of *measuring* rather than *mathematical instruments* in quantum physics:

> It will interest mathematical circles that the mathematical instruments created by the higher algebra play an essential part in the rational formulation of the new quantum mechanics. Thus, the general proofs of the conservation theorems in Heisenberg's theory carried out by Born and Jordan are based on the use of the theory of matrices, which go back to Cayley and were developed especially by Hermite. It is to be hoped that a new era of mutual stimulation of mechanics and mathematics has commenced. To the physicists it will at first seem deplorable that in atomic problems we have apparently met with such a limitation of our usual means of visualization. This regret will, however, have to give way to thankfulness that mathematics in this field, too, presents us with the tools to prepare the way for further progress. ([15], v. 1, p. 51)

The subsequent history has proved that Bohr was too optimistic as concerns the physicists' attitude. There has been 'thankfulness that mathematics in this field, too, presents us with the tools to prepare the way for further progress'. On the other hand, the discontent with 'the limitation' in question has never subsided and is still with us now. The precision of Bohr's statement is commendable: first, in stressing the essential role of the mathematics of QM, especially for the rigorous proof of the conservation theorems, and, second, in speaking of 'a new era of mutual stimulation of *mechanics* and mathematics', rather than *only* physics and mathematics (the statement applies also to these relationships). At stake are elemental *individual* quantum processes and events. The mathematical science, both representational and predictive, of these processes in classical physics is mechanics, which is now replaced by QM, as a non-realist theory that only predicts, in probabilistic or statistical terms, the effects (observed in measuring instruments) of such processes without representing them.

Although this elaboration may appear to announce a programme different from the one Bohr came to follow later, defined by his emphasis of the irreducible role of measuring instruments in quantum physics, by taking this view one would miss Bohr's understanding of the significance

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

of mathematics in QM. From 'Atomic theory and mechanics' on, Bohr always saw quantum physics as defined by the essential roles of both measuring and mathematical instruments, of experimental and mathematical technologies, in their reciprocal relationships. The very appeal to 'instruments' is hardly casual. Apart from the fact that such choices of expression are never casual in Bohr, the point is consistent with Bohr's view of mathematics, as also (although not only) a technology of thought, specifically in physics [69].

Experimental technology is a broader concept than 'measuring instruments', and it would, for example, involve devices that make it possible to apply the measuring instruments or other devices. More generally, technology is a means of doing something, and doing it more successfully than previously. The experimental technology of quantum physics enables us to understand how Nature works at the ultimate level of its constitution (in Bohr and the present view, via the effects of quantum objects on measuring devices), or obtaining and using information through quantum phenomena, which quantum objects enable us to have and manipulate, as in quantum cryptography and computing. Quantum objects themselves are not technology; they are something technology helps us to discover, understand, work with and so forth. However, they can become part of technology, beginning with the quantum parts of measuring instruments through which they interact with quantum objects concerning which we obtain information, or as part of the devices we use elsewhere, such as lasers, MRI machines and so forth.

In its use in physics, nearly all mathematics is technology. But one could think of the technological functioning and concepts of mathematics as operative even in mathematics itself, insofar as certain mathematical tools, such as homotopy or cohomology groups, are technologies akin to measuring instruments. According to J.-P. Marquis, who borrows the concept from physics and even specifically quantum physics, 'they provide information about the corresponding topological space … they are epistemologically radically from … *transformation* [symmetry] groups of a space. They do not act on anything. The purpose of these *geometric* devices is to classify spaces by their different *homotopy* [or cohomology] *types*'. By contrast, fibrations (through which homotopy and cohomology groups are defined) are not 'measuring instruments', but rather 'devices that make it possible to apply measuring instruments [such as cohomology and homotopy groups] and other devices' ([70], p. 259). In physics, symmetry groups are of course a mathematical technology. It follows, however, that, in parallel with experimental technology, the mathematical technology of physics or even of mathematics itself is not only or even primarily a way of representing reality but is also a means of experimenting with reality, and of creating new realities, physical and mathematical or philosophical. One might also see philosophy or even thinking in general as technology (although, again, not only as technology).[20] This subject, however, may lead one into 'metaphysical depths', which would require an engagement with philosophical works that would be difficult to undertake here. In commenting on some of the most basic concepts we use, H. Weyl once said:

> We cannot set out here in search of a definitive elucidation of what is to be a state of affairs, a judgment, an object, or a property. This task leads into metaphysical depths. And concerning it one must consult men, such as Fichte, whose names may not be mentioned among mathematicians without eliciting an indulgent smile. ([72], p. 7)

This is a cut-off that I must adopt here. Nevertheless, I would like to finish by citing M. Heidegger's conclusion in *The Question Concerning Technology and Other Essays* (which cites Heisenberg's article along the way) ([7], p. 23):

> There was a time when it was not technology [in its conventional sense] alone that bore the name *techne*. Once that revealing that brings forth truth into the splendor of radiant appearing also was called *techne*.

[20]Cf. an intriguing recent approach to representing sensation–perception dynamics in terms of quantum-like instruments in [71]. One might speak of 'circuits' in this context.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

> Once there was a time when the bringing forth of the true into the beautiful was called *techne*. And the *poiesis* of the fine arts also was called *techne*.
>
> In Greece, at the outset of the destining of the West, the arts soared to the supreme height of the revealing granted them... And art was simply called *techne*. It was a single, manifold revealing. It was ..., *promos*, i.e. yielding to the holding-sway and the safekeeping of truth.
>
> The arts were not derived from the artistic. Art works were not enjoyed aesthetically. Art was not a sector of cultural activity.
>
> What, then, was art – perhaps only for that brief but magnificent time? Why did art bear the modest name *techne*? Because it was a revealing that brought forth and hither, and therefore belonged within *poiesis*. It was finally that revealing which holds complete sway in all the fine arts, in poetry, and in everything poetical that obtained *poiesis* as its proper name...
>
> Whether art may be granted this highest possibility of its essence in the midst of the extreme danger [of modern technology], no one can tell. Yet we can be astounded. Before what? Before this other possibility: that the frenziedness of technology may entrench itself everywhere to such an extent that someday, throughout everything technological, the essence of technology may come to presence in the coming-to-pass of truth.
>
> Because the essence of technology is nothing technological, essential reflection upon technology and decisive confrontation with it must happen in a realm that is, on the one hand, akin to the essence of technology and, on the other, fundamentally different from it.
>
> Such a realm is art. But certainly only if reflection on art, for its part, does not shut its eyes to the constellation of truth after which we are *questioning*.
>
> Thus questioning, we bear witness to the crisis that in our sheer preoccupation with technology we do not yet experience the coming to presence of technology, that in our sheer aesthetic-mindedness we no longer guard and preserve the coming to presence of art. Yet the more questioningly we ponder the essence of technology, the more mysterious the essence of art becomes. ([7], pp. 34–35)

I would argue that the experimental and mathematical technology of quantum physics are *techne* in the sense that Heidegger wants to give the term here, and the digital technology of quantum physics may acquire this sense too. By the same token, fundamental physics, experimental and theoretical, 'guard and preserve the coming to presence of art', the art of physics and the art of thought.

Competing interests. The author declares that there are no competing interests.

Funding. I received no funding for this study.

Acknowledgements. I express my special thanks to G. Mauro D'Ariano, Lucien Hardy, Gregg Jaeger and Andrei Khrennikov for many productive discussions on the subjects considered here over many years, and D'Ariano and Hardy, additionally, for their help in clarifying the subjects discussed in §5. I also thank Laurent Freidel and Paolo Perinotti for helpful exchanges. Both anonymous referees of the article made valuable suggestions, which I gratefully acknowledge here. I am grateful to the Perimeter Institute for Theoretical Physics for the invitation, hosted by Lucien Hardy, to be a short-term visitor in November 2015, which was immensely helpful during the last stages of my work on this article, a version of which was presented at a seminar at the Institute.

References

1. Wikipedia. The Higgs boson. See https://en.wikipedia.org/wiki/Higgs_boson.
2. CERN. Accelerated science: images. See http://home.cern/images/tagged/Higgs-boson.
3. Peskin M, Schroeder D. 1995 *Introduction to quantum field theory*. Boulder, CO: Westview Press.
4. Heisenberg W. 1930 *The physical principles of the quantum theory*. [Transl. by K Eckhart, FC Hoyt.] New York, NY: Dover.
5. Plato. 2005 Phaedo. In *The collected dialogues of Plato* (eds E Hamilton, H Cairns), pp. 40–98. Princeton, NJ: Princeton University Press.
6. Kant I. 1997 *Critique of pure reason*. [Transl. by P Guyer, AW Wood.] Cambridge, UK: Cambridge University Press.

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

7. Heidegger M. 2004 *The question concerning technology and other essays*. New York, NY: Harper.
8. Pincock C. 2012 *Mathematics and scientific representation*. Oxford, UK: Oxford University Press.
9. Van Frassen B. 2008 *Scientific representation: paradoxes of perspective*. Oxford, UK: Oxford University Press.
10. Cartwright N. 1983 *How the laws of physics lie*. Oxford, UK: Oxford University Press.
11. Hacking I. 1983 *Representing and intervening: introductory topics in the philosophy of natural science*. Cambridge, UK: Cambridge University Press.
12. Cartwright N. 1999 *The dappled world: a study of the boundaries of science*. Cambridge, UK: Cambridge University Press.
13. Galison P. 1997 *Image and logic: a material culture of microphysics*. Chicago, IL: University of Chicago Press.
14. Latour B. 1999 *Pandora's hope: essays on the reality of science studies*. Cambridge, MA: Harvard University Press.
15. Bohr N. 1987 *The philosophical writings of Niels Bohr*, vols 1–3. Woodbridge, CT: OxBow Press.
16. Schrödinger E. 1935 The present situation in quantum mechanics (1935). In *Quantum theory and measurement* (eds JA Wheeler, WH Zurek), pp. 152–167. Princeton, NJ: Princeton University Press.
17. Khrennikov A. 2012 Quantum probabilities and violation of CHSH-inequality from classical random signals and threshold type detection scheme. *Prog. Theor. Phys.* **128**, 31–58. (doi:10.1143/PTP.128.31)
18. Plotnitsky A, Khrennikov A. 2015 Reality without realism: on the ontological and epistemological architecture of quantum mechanics. *Found. Phys.* **95**, 1269–1300. (doi:10.1007/s10701-015-9942-1)
19. D'Ariano GM, Manessi F, Perinotti P. 2014 Determinism without causality. *Phys. Scr*. **T163**, p014013-1–014013-8. (doi:10.1088/0031-8949/2014/T163/014013)
20. Plotnitsky A. 2011 Dark materials to create more worlds: on causality in classical physics, quantum physics, and nanophysics. *J. Comput. Theor. Nanosci*. **8**, 983–997. (doi:10.1166/jctn.2011.1778)
21. Pawlowski M, Paterek T, Kaszlikowski D, Scarani V, Winter A, Žukowski MZ. 2009 A new physical principle: information causality. *Nature* **461**, 1101–1104. (doi:10.1038/nature08400)
22. Hardy L. 2011 Foliable operational structures for general probabilistic theory. In *Deep beauty: understanding the quantum world through mathematical innovation* (ed. H Halvarson), pp. 409–442. Cambridge, UK: Cambridge University Press.
23. Gillies D. 2000 *Philosophical theories of probability*. London, UK: Routledge.
24. Háyek A. 2014 Interpretation of probability. In *Stanford encyclopedia of philosophy* (ed. EN Zalta). See http://plato.stanford.edu/archives/win2012/entries/probability-interpret/.
25. Khrennikov A. 2009 *Interpretations of probability*. Berlin, Germany: de Gruyter.
26. De Finetti B. 2008 *Philosophical lectures on probability*. [Transl. by H. Hosni.] Berlin, Germany: Springer.
27. Jaynes ET. 2003 *Probability theory: the logic of science*. Cambridge, UK: Cambridge University Press.
28. Bell JS. 2004 *Speakable and unspeakable in quantum mechanics*. Cambridge, UK: University Press.
29. Cushing JT, McMullin E (eds). 1989 *Philosophical consequences of quantum theory: reflections on Bell's theorem*. Notre Dame, IN: Notre Dame University Press.
30. Ellis J, Amati D (eds). 2000 *Quantum reflections*. Cambridge, UK: Cambridge University Press.
31. Brunner N, Gühne O, Huber M (eds). 2014 Special issue on 50 years of Bell's theorem. *J. Phys. A* **42**, 424024.
32. Heisenberg W. 1925 Quantum-theoretical re-interpretation of kinematical and mechanical relations. In *Sources of quantum mechanics* (ed. BL van der Warden), pp. 261–277. New York, NY: Dover.
33. Plotnitsky A. 2012 *Niels Bohr and complementarity: an introduction*. New York, NY: Springer.
34. D'Ariano GM, Perinotti P. 2014 Derivation of the Dirac equation from principles of information processing. *Phys. Rev. A* **90**, 062106. (doi:10.1103/PhysRevA.90.062106)
35. Bohr N. 1913 On the constitution of atoms and molecules (Part 1). *Philos. Mag*. **26**, 1–25. (doi:10.1080/14786441308634955)

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022

36. Dirac PAM. 1928 The quantum theory of the electron. *Proc. R. Soc. Lond. A* **117**, 610–624. (doi:10.1098/rspa.1928.0023)
37. Dirac PAM. 1962 Interview with T. Kuhn, April 1, 1962. Niels Bohr Archive, Copenhagen, Denmark, and American Institute of Physics, College Park, MD. See http://www.aip.org/history/ohilist/.
38. Pauli W. 1927 Zur Quantenmechanik des magnetischen Elektrons. *Z. Phys.* **43**, 601–623. (doi:10.1007/BF01397326)
39. Wigner EP. 1939 On unitary representations of the inhomoneneous Lorentz group. *Ann. Math.* **40**, 149–204. (doi:10.2307/1968551)
40. Plotnitsky A. 2015 A matter of principle: the principles of quantum theory, Dirac's equation, and quantum information. *Found. Phys.* **95**, 1222–1268. (doi:10.1007/s10701-015-9928-z)
41. Heisenberg W. 1989 *Encounters with Einstein, and other essays on people, places, and particles*. Princeton, NJ: Princeton University Press.
42. Weinberg S. 1996 What is an elementary particle? See http://www.slac.stanford.edu/pubs/beamline/27/1/27-1-weinberg.pdf.
43. Kuhlman M. 2014 Quantum field theory. In *Stanford encyclopedia of philosophy* (ed. E Zalta). See http://plato.stanford.edu/entries/quantum-field-theory/.
44. Weinberg S. 2005 *The quantum theory of fields.* Vol. 1. *Foundations*. Cambridge, UK: Cambridge University Press.
45. Teller P. 1995 *An interpretive introduction to quantum field theory*. Princeton, NJ: Princeton University Press.
46. Cao TY (ed.). 1999 *Conceptual foundations of quantum field theories*. Cambridge, UK: Cambridge University Press.
47. Schweber S. 1994 *QED and the men who made it: Dyson, Feynman, Schwinger, and Tomonaga*. Princeton, NJ: Princeton University Press.
48. Wilczek F. 2005 In search of symmetry lost. *Nature* **423**, 239–247. (doi:10.1038/nature03281)
49. Pais A. 1986 *Inward bound: of matter and forces in the physical world*. Oxford, UK: Oxford University Press.
50. Chiribella G, D'Ariano GM, Perinotti P. 2011 Informational derivation of quantum theory. *Phys. Rev. A* **84**, 012311-1-39. (doi:10.1103/PhysRevA.84.012311)
51. Fuchs CA. 2002 Quantum mechanics as quantum information (and only a little more). (http://arxiv.org/abs/quant-ph/0205039)
52. Hardy L. 2001 Quantum mechanics from five reasonable axioms. (http://arxiv.org/abs/quant-ph/0101012).
53. Jaeger G. 2007 *Quantum information: an overview*. New York, NY: Springer.
54. Mermin ND. 2007 *Quantum computer science: an introduction*. Cambridge, UK: Cambridge University Press.
55. Wheeler JA. 1990 Information, physics, quantum: the search for links. In *Complexity, entropy and the physics of information* (ed. WH Zurek), pp. 3–28. Redwood, CA: Addison-Wesley.
56. Fuchs C, Mermin ND, Schack R. 2013 An introduction to QBism with an application to the locality of quantum mechanics. (http://arxiv.org/abs/1311.5253)
57. Chiribella G, D'Ariano GM, Perinotti P. 2010 Probabilistic theories with purification. *Phys. Rev. A* **84**, 062348-1–062348-40. (doi:10.1103/PhysRevA.81.062348)
58. Schrödinger E. 1935 Discussion of probability relations between separated systems. *Proc. Camb. Phil. Soc.* **31**, 555–563. (doi:10.1017/S0305004100013554)
59. Bohr N. 1935 Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696–702. (doi:10.1103/PhysRev.48.696)
60. Dickson M. 2011 Non-relativistic quantum mechanics. In *Philosophy of physics: part A* (eds J Butterfield, J Earman), pp. 275–416. Amsterdam, The Netherlands: North Holland.
61. Schwinger J. 1988 Hermann Weyl and quantum kinematics. In *Exact sciences and their philosophical foundations* (eds W Depper, K Hübner). Frankfurt, Germany: Peter Lang.
62. Schwinger J. 2001 *Quantum mechanics: symbolism of atomic measurement*. New York, NY: Springer.
63. Jaeger G. 2016 Grounding the randomness of quantum measurement. *Phil. Trans. R. Soc. A* **374**, 20150238. (doi:10.1098/rstb.2015.0238)
64. Coecke B. 2010 Quantum picturalism. *Contemp. Phys.* **51**, 59–83. (doi:10.1080/00107510903257624)

65. Hardy L. 2013 Reconstructing quantum theory. (http://arxiv.org/abs/1303.1538)
66. Hardy L. 2010 A formalism-local framework for general probabilistic theories including quantum theory. (http://arxiv.org/abs/1005.5164)
67. Coecke B, Paquette EO. 2009 Categories for the practising physicist. (http://arxiv.org/abs/0905.3010)
68. Hardy L. 2007 Towards quantum gravity: a framework for probabilistic theories with non-fixed causal structure. *J. Phys. A* **40**, 3081–3099. (doi:10.1088/1751-8113/40/12/S12)
69. Bohr N. 1956 Mathematics and natural philosophy. In *The philosophical writings of Niels Bohr.* Vol. 4. *Causality and complementarity, supplementary papers* (eds J Faye, HJ Folse), pp. 164–169. Woodbridge, CT: OxBow Press.
70. Marquis JP. 2006 A path to epistemology of mathematics: homotopy theory. In *The architecture of modern mathematics: essays in history and philosophy* (eds J Ferreirós, J Gray), pp. 239–260. Oxford, UK: Oxford University Press.
71. Khrennikov A. 2015 Quantum-like modeling of cognition. *Front. Phys.* **22**. (doi:10.3389/fphy.2015.00077)
72. Weyl H. 1928 *The continuum: a critical examination of the foundation of analysis*. [Transl. by S Pollard, T Bole; rpt. 1994.] New York, NY: Dover.

rsta.royalsocietypublishing.org *Phil. Trans. R. Soc. A* **374**: 20150239

Downloaded from https://royalsocietypublishing.org/ on 03 March 2022