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Weak value controversy

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Opinion piece

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Recent controversy regarding the meaning and usefulness of weak values is reviewed. It is argued that in spite of recent statistical arguments by Ferrie and Combes, experiments with anomalous weak values provide useful amplification techniques for precision measurements of small effects in many realistic situations. The statistical nature of weak values is questioned. Although measuring weak values requires an ensemble, it is argued that the weak value, similarly to an eigenvalue, is a property of a single pre- and post-selected quantum system.

This article is part of the themed issue 'Second quantum revolution: foundational questions'.

1. Aharonov, Albert and Vaidman paper

The concept of the weak value was introduced almost 30 years ago by Aharonov, Albert and Vaidman (AAV) [1]. From the publication of the letter 'How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100' and until today it continues to be in the centre of a hot controversy. Here, I will review some of its controversial aspects and will clarify my point of view.

In [1], the weak value was defined as the outcome of the usual measuring procedure with weakened coupling performed on pre- and post-selected ensembles of quantum systems. The weakness condition was that the coupling does not change significantly the quantum state of the system. The concept was defined in the framework of the two-state vector formalism which describes pre- and post-selected systems by both forward and backward evolving quantum states, respectively. The measurement interaction has to be weak enough not to change significantly these states, where 'significantly' means that their scalar product remains approximately the same at all times during the measurement interaction.

It was shown that in the standard von Neumann model of measuring a variable A, in the limit of weak

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coupling, the quantum state of the pointer after the post-selection, $\Psi(q)$, is 'shifted' by the weak value A_w

$$\Psi(q) \rightarrow \Psi(q - A_w). \tag{1.1}$$

The reasons for the quotation marks are a missing proportionality factor between the value of A and the pointer variable q and a missing normalization factor which should appear when the weak value is a complex number. For a pre- and post-selected system described at time t by the two-state vector $\langle\phi|\,|\psi\rangle$ [2], the weak value is defined as

$$A_w \equiv \frac{\langle\phi|A|\psi\rangle}{\langle\phi\,|\,\psi\rangle}. \tag{1.2}$$

The real part of A_w, which is the average reading of the pointer of the standard measuring device, might be much larger than all eigenvalues of A.

Shortly after appearance of the AAV paper, Duck, Stevenson and Sudarshan wrote [3]: 'One's initial reaction is that this is impossible. This prejudice is reinforced when one finds that AAV's paper contains several errors'. Leggett and Peres published (critical) comments in *Physical Review Letters* [4,5]. They could not accept that a value associated with a physical variable A can be anything different from some eigenvalue of A. But continuation in [3] was: 'Nevertheless, after a careful study, we have concluded that AAV's main point does have validity' and after appearance of our reply to Peres & Leggett [6], Duck, Stevenson and Sudarshan added: 'A new manuscript by Aharonov & Vaidman [6] clarifies the mathematical example originally presented in ref. [1]. We refer the interested reader to this paper, and withdraw our earlier criticism of this example'.

In his comment, Leggett wrote: 'In a true measurement, by contrast, the measured value tells us much more than just the effect of the system on the measuring device'. It shows that this controversy is about semantics. We considered the main relevant thing regarding a physical variable of a system, is how it affects other systems. We stressed in the reply [6] that for 'any measuring procedure of a physical variable the coupling can be made weak enough such that the effective value of the variable for a preselected and postselected ensemble will be its weak value'. Moreover, since the result does not rest on the specific form of the interaction, it need not be a measurement interaction. The only requirement is the weakness of the interaction.

Any weak enough coupling to a variable A is an effective coupling to A_w. For a coupling to a continuous variable, we obtain the 'shift' (1.1), and more generally, for a weak coupling to any variable, in the interaction Hamiltonian the operator A can be replaced by the c-number A_w [7].

2. Amplification

The hope for practical applications of the weak measurement procedure was expressed in the conclusions of [1] which pointed out that with small scalar product $\langle\phi\,|\,\psi\rangle$ we get 'tremendous amplification' of small effects. The first experiment [8] showed a factor of 20. It was the work of Hosten & Kwiat [9] 20 years later that used the amplification effect for observing the spin Hall effect for light, which brought the AAV amplification scheme to the centre of current research. This tiny effect had not been observed before by any other means. Shortly after, weak value measurement techniques allowed Dixon *et al.* [10] to measure an unimaginably small rotation of a mirror and many more implementations of the AAV method were reported [11–17].

This activity brought a new controversy. Ferry and Combes (FC) posted a preprint titled 'Weak values considered harmful' [18]. Robert Garisto, the Editor of *Physical Review Letters* chose this submission as particularly interesting, but softened its title: 'Weak value amplification is suboptimal for estimation and detection' [19]. Numerous works were published praising and criticizing the AAV scheme as a method for parameter estimation [20–41]. Indeed, theoretical analysis is complicated and depends on the models of noises in the experiment. However, I find that most of these sophisticated analyses are unnecessary for explaining why, in spite of the 'statistically rigorous arguments' of Ferrie & Combes [19], the AAV amplification scheme was useful in numerous experiments [42]. The explanation is that the assumptions in their statistical analysis are irrelevant for many realistic experimental situations.

I found the main erroneous assumption which led Ferrie and Combes to their incorrect conclusions thanks to my direct involvement in weak measurement experiments [12,43]. The limiting factor in these and other experiments is not the number of pre-selected quantum systems (photons) considered by Ferrie and Combes, but the number of detected, post-selected photons. The saturation of the detectors generally happens much before the power limitation of the laser source kicks in. Thus, the low probability of the post-seletction, the main negative factor in experiments with anomalously large weak values, is not relevant. In fact, I also have been involved in a weak value measurement experiment recycling photons which were not post-selected [44], but the results only convinced me that it is an unnecessary complication of the experiment.

The argument made in my comment [42] was recently developed by Harris, Boyd and Lundeen in *Physical Review Letters*: 'Weak value amplification can outperform conventional measurement in the presence of detector saturation' [45]. Ironically, their letter includes the statement 'saturation alone does not confer an advantage to the WVA' which is technically valid due to their assumption of a specific saturation model and unrealistic ideal noiseless situation. The main point of their letter was what I stated in my comment: Ferrie and Combes' calculations, as well as a few other similar results, are not relevant for real experiments.

3. Classical analogue to weak value

Ferrie and Combes took the controversy about anomalous weak values even further, publishing another letter which obtained Editor's Suggestions distinction [46]. The letter has a provocative title: 'How the result of a single coin toss can turn out to be 100 heads'. In this letter, FC claimed to show 'that weak values are not inherently quantum, but rather a purely statistical feature of pre- and post-selection with disturbance'. To prove their point, they presented a purely classical situation with a coin toss which is supposed to be analogous to the example presented in the first publication of the weak value which has the title: 'How the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100' [1].

I argued that the analogy is an illusion [47]. The weak value of a variable of a system is defined by pre- and post-selected states of the system. The weak value of 100 for the spin z component of a particle appeared for the particular pre- and post-selected spin states:

$$\left.\begin{aligned} |\psi\rangle &= \cos\frac{\alpha}{2}|\uparrow_x\rangle + \sin\frac{\alpha}{2}|\downarrow_x\rangle, \quad \tan\frac{\alpha}{2} = 100 \\ \text{and} \quad |\phi\rangle &= |\uparrow_x\rangle. \end{aligned}\right\} \tag{3.1}$$

The number '100' appears due to the almost opposite directions of pre- and post-selected spins and specified by the parameter α of the pre-selected state. It does not depend on a particular disturbance of the measurement: every weak enough coupling to the spin will show $(\sigma_z)_{\rm w} = 100$. The disturbance of the measurement might distort the weak value, it does not specify it. The number 100 is obtained in the limit of vanishing disturbance.

By contrast, in the example of FC, the initial state is '1' and the final state is '−1', their classical system does not have enough complexity to define different numbers. There are only four possible pre- and post-selections, so we can get only four possible 'weak values' of a given variable. FC got the value 100 by playing with the definition of disturbance in their 'weak' measurement. They could equally well get value 1000 for the same pre- and post-selection. There is nothing in their construction analogous to (1.2) that provides a functional dependence on the pre- and post-selected states of the system. The quantum weak value of a variable with discrete eigenvalues can obtain continuous spectrum of values due to a continuum of possible pre-selected states and a continuum of possible post-selected states. The continuum of classical 'analogue of weak values' is obtained by tailoring the measurement interaction. The difference between AAV and FC is not just quantum versus classical; the set-ups are conceptually different, so there cannot be an analogy between the two cases.

Apart from my comment [47] on the second letter of FC, there were many more [48–54]. *Physical Review Letters* chose to publish only the comment of Brodutch [51]. Apparently, it was the most convenient choice for FC. It pointed out that the classical model of the FC letter had a technical mistake: a measurement of a variable that can have values ±1 could not yield 100. Probably, the best reply of FC would be: sure, Brodutch is right, the measurement procedure is not legitimate, but the error is exactly the same as in the AAV quantum measurement procedure!

FC in their reply [55], instead, made again the connection to the work of Garretson *et al.* [56] (based on [57]) discussing 'weak-valued probability distribution of momentum transfer' in which-way experiments. The term 'weak value of probability' has rigorous definition as the weak value of the projection operator. In this experiment, a weak value of a projection operator on a particular momentum was measured. The important difference in this work relative to the weak value of a standard weak measurement is the presence of additional which-way measurement, not related to the weak measurement of momentum. It is this additional which-way measurement which caused the disturbance. The disturbance does not go to zero with the limit of vanishing coupling of the weak measurement. Apparently, the analogy to this disturbance led to '100 heads' in the FC example. The number 100 did not come from pre- and post-selection of the coin. See more analysis of disturbance in weak measurements in [58,59].

The AAV experiment can be explained as an interference of classical electromagnetic waves. This is a generic feature of weak measurement experiments in which the measuring device is a degree of freedom of the pre- and post-selected particle itself. I doubt that a correct classical *statistical* analogy of the AAV experiment exists, especially if the challenging modification of the weak measurement using external measuring device, like in [17], is considered. However, it at least can be formulated when we consider a toss of a real classical coin with pre- and post-selected states specified by their actual orientation in space; compare with a recent proposal [60]. The space of pre- and post-selected states then is rich enough for a functional relation similar to (1.2).

4. Weak value as a contextual value

Another line of argument against FC's claim was the result by Pusey [61] who showed that anomalous weak values constitute proofs of the incompatibility of quantum theory with non-contextual ontological models [62]. This result has recently been demonstrated experimentally [63]. The connection between weak values and contextuality was pointed out by Dressel *et al.* [64,65] who introduced 'contextual values' and viewed the weak value as an example of a contextual value. In [64], they write: 'The idea behind contextual value stems from the observation that the intrinsically measurable quantities in the quantum theory are the outcome probabilities for a particular measurement setup'.

Although the conclusion of this approach is what I strongly believe: anomalous weak values cannot be explained in the framework of classical statistical theory, I am very far from accepting the connection between contextual values and weak values. At the price of accepting parallel worlds, I view quantum theory as a deterministic theory [66]. And I disagree that it is based on outcome probabilities: apart from predicting (the illusion of) probabilities for outcomes of experiments, quantum mechanics makes numerous definite predictions: spectrum of atoms, etc.

If we insist on considering probabilistic theories, then there is a way to introduce weak values in a classical theory, but a very specific one, a classical theory with an epistemic restriction [67]. In this theory, as the authors point out, 'anomalous weak values do not appear in our analysis, as all observables in our model possess an unbounded spectrum. Consistent with the results of [61], our model is also noncontextual: the ERL [epistemically restricted Liouville] mechanics provides an explicit noncontextual ontological model for all procedures described here'. Thus, this theory cannot help to resolve the controversy.

Contextual value techniques lead to a natural definition of a general conditioned average that converges uniquely to the quantum weak value in the minimal disturbance limit. And I find it of interest that it helps to answer the criticism of Parrott [68,69]. However, I have another argument which allows one to avoid dealing with statistics. In the next section, I will argue that the weak value can be considered beyond its statistical meaning. And if the nature of weak values is not statistical, then statistical analyses are not relevant.

5. Weak value as a property of a single system

Recently, together with Harald Weinfurter and his group in Munich, we brought another theoretical argument demonstrated in an actual experiment which refutes any attempt to find a classical statistical analogue of the weak value [70]. The argument is that the weak value is a property of a single pre- and post-selected quantum system. Thus, it cannot have an analogy as a statistical property of an ensemble.

Weak values were introduced as outcomes of weak measurements [1], which have large uncertainty in the pointer position. Thus, in experiments, the weak value is obtained as a statistical average of the pointer readings. Even among proponents of this concept, the weak value is frequently understood as a mere generalization of the expectation value for the case when the quantum system is post-selected, i.e. a conditional expectation value [71,72].

Contrary to the classical case, if we are given a single system in which a quantum variable has a known definite eigenvalue, we cannot test this fact with certainty. We cannot distinguish with certainty between this situation and a case in which we started in a superposition which included this eigenvalue. Still there is some certainty about this situation: we know that a measurement of this variable will yield the eigenvalue with certainty. This is what makes this situation not statistical. This is not the case if we are given a quantum system described by a known expectation value of the variable. There is no single experiment with definite outcome specified by the expectation value.

What we have found in [70] is that when the system is described by a weak value, it interacts with other systems almost identically to the system described by a numerically equal eigenvalue, but significantly different from the case of a numerically equal expectation value.

We compare the operational meaning of a weak value, an eigenvalue and an expectation value. While the definition of the weak value is based on the pure two-state vector with states $|\psi\rangle$ and $|\varphi\rangle$ considered at a particular time t, its operational meaning relies on interactions with other systems which create entanglement. The way to deal with this problem is to consider a short period of time around time t to evaluate the action of the system on other systems. During this small time, the entanglement can be considered negligible, but then the effect is very small too. To observe it we need an ensemble. We attribute properties to each system individually, but for testing our claim we will use an ensemble.

We consider a standard measuring procedure described by interaction Hamiltonian $H_{\rm int} = gAP$. We assume that at time $t = 0$, the system was prepared in state $|\psi\rangle$ and shortly after, at time $t = \epsilon$, was found in state $|\varphi\rangle$. The pointer at time $t = 0$ is in a Gaussian state Φ_0. For a comparison of different cases, we consider the pointer state at time $t = \epsilon$, after the interaction with the integer spin observable $A \equiv \sum_j j|j\rangle\langle j|$. If the spin state is the eigenstate $|1\rangle$, i.e. the variable has the eigenvalue $A = 1$, then at time $t = \epsilon$, independently of the result of the post-selection measurement, the pointer state is shifted:

$$\Phi_{\rm e} = \mathcal{N}\, {\rm e}^{-(Q-g\epsilon)^2/4\Delta^2}. \tag{5.1}$$

We compare three cases with the same numerical value: the eigenvalue, the weak value, and the expectation value. We evaluate the effect of the interaction by calculating the distance between quantum states expressed by the Bures angle. Other metrics can be used too, but the advantage of the Bures angle is that in the limit, it is proportional to time in a measurement-type evolution. The distance between the initial state of the measuring device Φ_0 and the final state (5.1) is

$$D_A(\Phi_0, \Phi_{\rm e}) \equiv \arccos|\langle\Phi_0\,|\,\Phi_{\rm e}\rangle| = \frac{g\epsilon}{2\Delta} + \mathcal{O}(\epsilon^3). \tag{5.2}$$

Consider now a pre- and post-selected system with $A_w = 1$, but in which both pre-selection and post-selection do not include the eigenstate $|1\rangle$. A two-state vector which provides this weak value is

$$\langle\varphi|\,|\psi\rangle = \frac{1}{\sqrt{5}}(\langle -1| - 2\langle 0|)\frac{1}{\sqrt{2}}(|-1\rangle + |0\rangle). \tag{5.3}$$

After the post-selection, the state of the pointer variable is

$$\Phi_w = \mathcal{N}(\epsilon)(2\,e^{-Q^2/4\Delta^2} - e^{-(Q+g\epsilon)^2/4\Delta^2}) \approx \mathcal{N}'(\epsilon)\,e^{-Q^2g^2\epsilon^2/4\Delta^4}\Phi_e. \tag{5.4}$$

Φ_w is effectively a Gaussian centred around $A_w = 1$ and is thus very close to Φ_e as seen from the Bures angle

$$D_A(\Phi_e, \Phi_w) = \frac{g^2\epsilon^2}{2\sqrt{2}\Delta^2} + \mathcal{O}(\epsilon^4). \tag{5.5}$$

The characteristic distance between states after the interaction for the time ϵ is approximately $g\epsilon/2\Delta$, so when the additional distance is proportional to ϵ^2, it can be neglected. Thus, in the limit of short interaction times, the pre- and post-selected systems with some weak value interact with other systems in the same manner as a system pre-selected in an eigenstate with a numerically equal eigenvalue. Not only are the expectation values of the positions of the pointers essentially the same, but also the full quantum states of the pointers are almost identical.

The situation changes considerably when the system is only pre-selected in a state with the expectation value $\langle A\rangle = 1$, which, however, is not the eigenstate $|1\rangle$. To show this, assume that the particle is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle). \tag{5.6}$$

At time $t = \epsilon$, now without post-selection, the pointer system is not described by a pure state, but by a mixture. A straightforward calculation yields the density matrix describing this mixture

$$\rho_{ex} = \frac{1}{2\sqrt{2\pi}\,\Delta}(e^{-(Q^2+Q'^2)/4\Delta^2} + e^{-((Q-2g\epsilon)^2+(Q'-2g\epsilon)^2)/4\Delta^2}). \tag{5.7}$$

The distance between ρ_{ex} and Φ_e, the state of the pointer after coupling to an eigenvalue, is

$$D_A(\Phi_e, \rho_{ex}) \equiv \arccos\left(\sqrt{\langle\Phi_e|\rho_{ex}|\Phi_e\rangle}\right) = \frac{g\epsilon}{2\Delta} + \mathcal{O}(\epsilon^3). \tag{5.8}$$

This is a significantly larger distance than (5.5). In fact, the distance (5.8) is of the same order as (5.2) and cannot be neglected for small ϵ.

While the pointer states (5.4) and (5.7) for a small enough ϵ correspond to similar probability distributions, they are fundamentally different. As in the case of an eigenstate (5.1), the final pointer state (5.4) corresponds to a shift of the original distribution given by a single number, the weak value. When the system is prepared in a superposition of eigenstates, the result is a mixture of two independent pointer distributions centred around the values 0 and 2, which cannot be described by a single parameter anymore.

Our demonstration of the weak value as a property of a single pre- and post-selected system shows that recent classical statistical analogies of weak values [46] which can be formulated only given an ensemble are artificial.

6. Conclusion

Quantum theory is about century old, but its foundations are still under hot debate. The quantum phenomena are very different from the classical picture, so there was a tendency in the early days to accept that quantum reality is not understandable in principle, and some questions should not be asked. One such question is the description of a pre- and post-selected quantum system. The weak value is a property of such a system and the standard formalism, without a backward evolving quantum state, lacks this concept. It is not that we cannot discuss the interaction of a pre- and post-selected system in the standard formalism, it is just much more difficult and lacks a transparent picture, since it has to involve entanglement with the other systems.

As one can see from the unusually long reference list which mostly consists of papers claiming contradictory statements, we are far from reaching a consensus about the meaning of weak values, and there are several other related controversies. What is the status of counterfactual statements about pre- and post-selected quantum systems [73,74]? What can be said about the past of quantum particles [75]? (I do not present the list of relevant references here.) Can weak values be measured strongly [32,76]?

In this paper, I just covered the controversies raised by the two letters of Ferrie & Combes [19, 46]. Although, as a referee, I think that these letters should have never been published (not to mention receive distinctions), in retrospect I admit that the controversy they raised brought deeper understanding of quantum mechanics and that this controversy can be considered as a part of a second quantum revolution. Our discovery that the weak value is more like an eigenvalue than an expectation value and that it is a property of a single system and not of an ensemble is far from being accepted, so the revolution is still in progress.

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